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THE MARCH MEETING IN NEW YORK

The two hundred ninety-seventh regular meeting of the American Mathematical Society was held at Columbia University, on Friday and Saturday, March 25–26, 1932. The attendance included the following one hundred sixty members of the Society:

C. R. Adams, R. B. Adams, R. P. Agnew, R. L. Anderson, R. C. Archibald, R. G. Archibald, J. L. Barnes, E. F. Beckenbach, A. C. Berry, G. D. Birkhoff, H. W. Bode, H. F. Bohnenblust, A. B. Brown, B. H. Brown, J. H. Bushey, S. S. Cairns, A. D. Campbell, P. A. Caris, W. B. Carver, W. F. Cheney, Alonzo Church, J. A. Clark, J. M. Clarkson, George Comenetz, R. P. Conkling, E. H. Cutler, Tobias Dantzig, J. L. Doob, Paul Dorweiler, Jesse Douglas, Arnold Dresden, J. K. Dyer, L. P. Eisenhart, H. T. Engstrom, C. B. Fisher, T. S. Fiske, W. B. Fite, D. A. Flanders, Edward Fleisher, Tomlinson Fort, R. M. Foster, C. A. Garabedian, J. J. Gergen, B. P. Gill, R. E. Gilman, Eli Gourin, M. C. Gray, Laura Guggenbühl, F. C. Hall, Marshall Hall, G. A. Hedlund, Einar Hille, Lulu Hofmann, G. M. Hopper, I. O. Horsfall, Harold Hotelling, W. G. Hubert, J. C. Hughes, W. A. Hurwitz, L. W. Hussey, Nathan Jacobson, R. L. Jeffery, R. A. Johnson, F. E. Johnston, Edward Kasner, O. D. Kellogg, L. S. Kennison, M. V. Kenny, B. F. Kimball, J. R. Kline, P. A. Knedler, Mark Kormes, H. L. Krall, J. H. Kusner, K. W. Lamson, A. W. Landers, V. V. Latshaw, W. S. Lawton, Solomon Lefschetz, M. M. Lehr, E. O. McCormick, N. H. McCoy, L. A. MacColl, H. F. MacNeish, L. C. Mathewson, F. M. Mears, A. E. Meder, E. R. C. Miles, A. K. Mitchell, H. H. Mitchell, U. G. Mitchell, E. C. Molina, Morris Monsky, C. N. Moore, C. B. Morrey, G. W. Mullins, V. F. Murray, S. B. Myers, C. A. Nelson, Philip Newman, D. P. Nile, Oystein Ore, F. W. Owens, G. A. Pfeiffer, Hillel Poritsky, E. L. Post, W. C. Randels, G. E. Raynor, L. M. Reagen, H. W. Reddick, M. S. Rees, R. G. D. Richardson, J. F. Ritt, J. H. Roberts, H. P. Robertson, S. A. Schelkunoff, H. M. Schlauch, C. E. Seely, I. M. Sheffer, C. G. Shover, L. P. Siceloff, L. G. Simons, James Singer, Abraham Sinkov, L. L. Smail, A. H. Smith, D. E. Smith, P. A. Smith, M. H. Stone, H. G. Swain, J. D. Tamarkin, Alexander Tartler, J. H. Taylor, J. M. Thomas, T. Y. Thomas, E. W. Titt, M. M. Torrey, A. W. Tucker, J. L. Vanderslice, I. L. Van Dyck, G. P. Wadsworth, R. J. Walker, Warren Weaver, Louis Weisner, E. H. Wells, M. E. Wells, Anna Pell Wheeler, H. S. White, Hassler Whitney, S. S. Wilks, John Williamson, W. A. Wilson, Clement Winston, H. P. Wirth, D. W. Woodard, H. M. Wright, R. C. Yates, Jacob Yerushalmy, Oscar Zariski, Leo Zippin.

No meeting of the Council of the Society was held. The Board of Trustees met on Friday evening at the home of Professor D. E. Smith.

The meeting opened on Friday morning with two sectional sessions, Analysis Situs and Points Sets, and Algebra and Arithmetic. At the end of the Friday morning sessions, Professor W. A. Hurwitz delivered an address on *General theorems on linear transformations of sequences*, with discussion led by Professor C. N. Moore and Dr. R. P. Agnew. The Friday afternoon session was opened with an address on *Summation of Fourier series*, by Professor Einar Hille, with discussion led by Professor J. D. Tamarkin and Dr. J. J. Gergen, followed by a general session for short papers. At the beginning of the Saturday morning session, Professor R. Courant, of the University of Göttingen, delivered an address on *Functional methods and characteristic values in the calculus of probabilities*. This was followed by sectional sessions, for Analysis and Geometry.

Titles and cross references to the papers (other than invited addresses) read at this meeting follow below; papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 13 were read before the section of Analysis Situs and Point Sets, Professor W. A. Wilson presiding; those numbered 14 to 23 before the section of Algebra and Arithmetic, Professor Fort presiding; those numbered 24 to 41 at the general session on Friday afternoon, President Eisenhart presiding; those numbered 42 to 51 before the section of Analysis, Professor Kellogg presiding; those numbered 52 to 61 before the section of Geometry, Professor R. A. Johnson presiding. President Eisenhart presided when the addresses were delivered. Professor Cummings' paper was read by Professor H. S. White. Mr. Hodge was introduced by Professor Lefschetz, Mr. Sherman by Professor Shohat, and Professor Zygmund by Professor Tamarkin.

1. *Concerning spaces which are uniordered relative to systems of closed and compact point sets*, by Professor J. H. Roberts. (Abstract No. 38–3–60.)

2. *On continuous curves with cyclic connection of higher order*, by Mr. J. H. Kusner. (Abstract No. 38–3–79.)

3. *On the Rutt-Nöbeling theorem*, by Dr. Leo Zippin (National Research Fellow). (Abstract No. 38–5–119.)

4. *Irreducible continuous curves*, by Dr. Leo Zippin (National Research Fellow). (Abstract No. 38–3–80–*t*.)

5. *On Morse's duality relations for manifolds*, by Dr. A. B. Brown. (Abstract No. 38–3–81.)

6. *Regular families of curves.* I, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–3–82.)

7. *A topological theorem on algebroid singularities*, by Dr. Oscar Zariski. (Abstract No. 38–5–120.)

8. *Concerning regular pseudo d-cyclic sets*, by Dr. L. M. Blumenthal. (Abstract No. 38–3–78–*t*.)

9. *Regular families of curves.* II, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 38–3–83–*t*.)

10. *k-dimensional sets in n-dimensional space*, by Professor R. L. Jeffery. (Abstract No. 38–1–57–*t*.)

11. *A certain transformation on metric spaces*, by G. T. Whyburn. (Abstract No. 38–3–66–*t*.)

12. *A relation between metric and euclidean spaces*, by Professor W. A. Wilson. (Abstract No. 38–3–84.)

13. *On angles in metric spaces*, by Professor W. A. Wilson. (Abstract No. 38–3–85–*t*.)

14. *Families of groups generated by two operators of the same order*, by Mr. Abraham Sinkov. (Abstract No. 38–3–86.)

15. *Significance of quadratic residues in factorization*, by Mr. Marshall Hall. (Abstract No. 38–3–87.)

16. *Non-commutative polynomials*, by Professor Oystein Ore. (Abstract No. 38–3–88.)

17. *The application of Bernoulli functions of negative order to differencing*, by Dr. B. F. Kimball. (Abstract No. 38–3–94.)

18. *On the factorization of divisors of zero in finite commutative rings*, by Dr. J. L. Dorroh (National Research Fellow). (Abstract No. 38–3–89–*t*.)

19. *On factorable polynomials in several indeterminates over a Galois field*, by Dr. Leonard Carlitz (International Research Fellow.) (Abstract No. 38–3–90–*t*.)

20. *On an arithmetical functional equation*, by Dr. Leonard Carlitz (International Research Fellow). (Abstract No. 38–3–91–*t*.)

21. *On a problem in additive arithmetic.* Second paper, by Dr. Leonard Carlitz (International Research Fellow). (Abstract No. 38–3–92–*t*.)

22. *On the approximate solution of certain equations*, by Professor Raymond Garver. (Abstract No. 38–3–93–*t*.)

23. *A generalized boundary value problem for the heat equation*, by Mr. F. G. Dressel. (Abstract No. 38–5–121–*t*.)

24. *On certain sufficient conditions that a set of constants should be Fourier constants*, by Professor C. N. Moore. (Abstract No. 38–3–95.)

25. *Successive curvatures of dynamical trajectories*, by Professor Edward Kasner. (Abstract No. 38–3–96.)

26. *Regular differential systems of the first order*, by Professor J. M. Thomas. (Abstract No. 38–3–97.)

27. *Applications of a covariant differentiation process*, by Professor H. V. Craig. (Abstract No. 38–3–101–*t*.)

28. *The boundary values of analytic functions*, by Dr. J. L. Doob. (Abstract No. 38–3–102–*t*.)

29. *Convergence criteria for double Fourier series*, by Dr. J. J. Gergen. (Abstract No. 38–3–103–*t*.)

30. *Note on homogeneous functionals*, by Dr. L. S. Kennison. (Abstract No. 38–3–59–*t*.)

31. *The inverse matrix for de la Vallée-Poussin summation*, by Professor George Rutledge. (Abstract No. 38–3–104–*t*.)

32. *A reliable method of obtaining the derivative function from smoothed data of observation*, by Professor George Rutledge. (Abstract No. 38–3–105–*t*.)

33. *An expansion of meromorphic functions*, by Professor J. L. Walsh. (Abstract No. 38–3–61–*t*.)

34. *Interpolation and functions analytic interior to the unit circle*, by Professor J. L. Walsh. (Abstract No. 38–3–62–*t*.)

35. *Note on the degree of convergence of sequences of analytic functions*, by Professor J. L. Walsh. (Abstract No. 38–3–63–*t*.)

36. *On the overconvergence of sequences of rational functions*, by Professor J. L. Walsh. (Abstract No. 38–3–64–*t*.)

37. *On polynomial interpolation to analytic functions with singularities*, by Professor J. L. Walsh. (Abstract No. 38–3–65–*t*.)

38. *Estimates, moments, and distributions of certain statistics in fragmentary samples*, by Dr. S. S. Wilks (National Research Fellow). (Abstract No. 38–3–106–*t*.)

39. *On the inversion of a special cyclic system*, by Professor Malcolm Foster. (Abstract No. 38–3–117–*t*.)

40. *On planar element strips*, by Professor P. F. Smith. (Abstract No. 38–3–118–*t*.)

41. *On lacunary trigonometric series*, by Professor Antoni Zygmund. (Abstract No. 38–5–122–*t*.)

42. *Sequences of meromorphic functions*, by Dr. J. L. Doob. (Abstract No. 38–3–107.)

43. *The Dirichlet problem for harmonic functionals*, by Mr. W. V. D. Hodge. (Abstract No. 38–3–108.)

44. *On the zeros of certain polynomials related to Laguerre and Jacobi polynomials*, by Mr. W. S. Lawton. (Abstract No. 38–3–109.)

45. *Elementary transformations*, by Mr. E. R. Lorch. (Abstract No. 38–1–56–*t*.)

46. *Sufficient conditions in the problem of the calculus of variations in n-space, in parametric form and under general end conditions*, by Mr. S. B. Myers. (Abstract No. 38–3–110.)

47. *On the numerators of the convergents of the Stieltjes continued fractions*, by Mr. Jacob Sherman. (Abstract No. 38–3–111.)

48. *A class of solutions of the heat equation*, by Mr. F. G. Dressel and Professor E. R. C. Miles. (Abstract No. 38–5–123.)

49. *On a matrix differential operator*, by Professor A. K. Mitchell. (Abstract 38–3–98.)

50. *Three theorems applicable to vibration theory*, by Professor B. F. Kimball. (Abstract No. 38–3–99.)

51. *Certain generalizations in the analysis of variance*, by Dr. S. S. Wilks (National Research Fellow). (Abstract No. 38–3–100.)

52. *Equiareal maps with parabolic meridians and parallels*, by Professor B. H. Brown. (Abstract No. 38–3–112.)

53. *The limit of the ratio of arc to chord for the curves drawn at a point on a surface*, by Mr. George Comenetz. (Abstract No. 38–3–113.)

54. *Conformal transformations in function space*, by Dr. L. S. Kennison. (Abstract No. 38–3–58.)

55. *The direction cosines of a p-plane, L_p, in a euclidean n-space, S_n*, by Dr. S. S. Cairns. (Abstract No. 38–3–114.)

56. *Some involutorial line transformations interpreted as points of V_2 of S_5*, by Mr. J. M. Clarkson. (Abstract No. 38–3–115.)

57. *Heptagonal systems of eight lines in a plane*, by Professor Louise D. Cummings. (Abstract No. 38–3–116.)

58. *Plane Cremona transformations arising from a generalization of the transformation by reciprocal radii*, by Dr. H. C. Shaub. (Abstract No. 38–1–22–*t*.)

59. *On summability of double sequences*, by Dr. R. P. Agnew (National Research Fellow). (Abstract No. 38–5–124–*t*.)

60. *Definition of a field by four postulates*, by Professor W. A. Hurwitz. (Abstract No. 38–5–125–*t*.)

61. *Characterization of collineations on the real projective plane.* Preliminary report, by Mr. Walter Prenowitz. (Abstract No. 38–5–126–*t*.)

TOMLINSON FORT,
Associate Secretary

THE APRIL MEETING IN CHICAGO

The two hundred ninety-eighth regular meeting of the Society was held at the University of Chicago on Friday and Saturday, April 8–9, 1932. About one hundred thirty persons attended the meetings, among whom were the following one hundred six members of the Society:

A. A. Albert, F. L. Axen, W. L. Ayres, R. P. Baker, R. W. Barnard, I. A. Barnett, Walter Bartky, W. D. Baten, S. F. Bibb, G. A. Bliss, Henry Blumberg, D. G. Bourgin, J. W. Bower, R. C. Bullock, R. S. Burington, L. E. Bush, R. D. Carmichael, E. W. Chittenden, A. B. Coble, E. G. H. Comfort, H. H. Conwell, A. T. Craig, H. B. Curry, D. R. Curtiss, L. E. Dickson, R. M. Ely, Arnold Emch, H. P. Evans, H. S. Everett, Gordon Fuller, H. L. Garabedian, Cornelius Gouwens, L. M. Graves, L. W. Griffiths, M. L. Hartung, O. C. Hazlett, M. R. Hestenes, E. H. Hildebrandt, T. H. Hildebrandt, T. F. Holgate, D. L. Holl, M. H. Ingraham, W. C. Krathwohl, A. C. Ladner, E. P. Lane, R. E. Langer, M. I. Logsdon, R. E. Lowney, W. A. Luby, A. C. Lunn, E. B. Lytle, J. S. McNair, C. C. MacDuffee, W. D. MacMillan, W. A. Manning, Morris Marden, A. E. May, J. R. Mayor, E. W. Miller, E. H. Moore, T. W. Moore, E. J. Moulton, F. H. Murray, Rufus Oldenburger, E. J. Olson, G. A. Parkinson, E. D. Pepper, T. S. Peterson, H. H. Pixley, Tibor Radó, G. Y. Rainich, E. A. Rasor, W. T. Reid, H. L. Rietz, P. G. Robinson, W. E. Roth, R. G. Sanger, I. J. Schoenberg, Henry Schultz, Nathan Schwid, H. A. Simmons, E. B. Skinner, H. E. Slaught, Burke Smith, E. S. Sokolinkoff, I. S. Sokolnikoff, A. A. Stafford, R. C. Stephens, P. M. Swingle, V. B. Teach, W. J. Trjitzinsky, P. L. Trump, J. S. Turner, H. L. Turrittin, A. M. Tuttle, J. I. Vass, H. S. Wall, L. E. Ward, K. W. Wegner, R. L. Wilder, F. B. Wiley, K. P. Williams, Y. K. Wong, F. E. Wood, Roscoe Woods, C. R. Worth.

A meeting of the Council was held on Friday afternoon, April 8, in Room 316, Eckhart Hall of the University of Chicago. Associate Secretary Ingraham announced the election of the following persons to membership in the Society:

Professor Scott Buchanan, University of Virginia;
Professor Iris Callaway, Georgia State Teachers College;
Mr. Henry Miot Cox, Georgia School of Technology;
Mr. George Franklin Cramer, Tulane University;
Mr. Merrill Meeks Flood, Princeton University;
Mr. George Frecheville, Agricultural Economics Research Institute, University of Oxford;
Mr. Nathan Jacobson, Princeton University;
Dr. Trinidad J. Jaramillo, Northern Luzon Junior College, Vigan, Philippine Islands;
Mr. Reinhardt Lunde Korgen, Bowdoin College;
Professor Nicolas Kryloff, Kieff, Ukraine, U. S. S. R.;
Mr. Enrique Linares, Central Board of Roads, Panama City;
Professor James Anson McLaughlin, St. Bonaventure's College;
Professor Walter Priest Morse, Ricker Junior College;
Mr. William Lind Morris, Gulf Research Laboratory, Pittsburgh;
Mr. Eugene Purdy Northrop, Yale University;
Mr. William Conrad Randels, Princeton University;
Dr. Max Goy Scherberg, University of Minnesota;
Mr. Jacob Sherman, League Island, Philadelphia;
Mr. Edward A. Shonka, St. Procopius College;
Mrs. Irene Fern Smith, Wilmington, Del.;
Mr. Jacob Reiff Kob Stauffer, University of Chicago;
Mr. Edward Warren Titt, Princeton University;
Mr. Orlin Edward Walder, South Dakota State College.

As Nominees of Allyn and Bacon:

Mr. Reece Croxton Blackwell, Johns Hopkins University;
Mr. George Proctor Wadsworth, Massachusetts Institute of Technology.

As Nominees of the National Life Insurance Company of the United States of America:

Mr. John F. Randolph, Cornell University;
Miss Ida Lucy van Dyck, Brown University.

As Nominees of Members of the Department of Mathematics of the George Washington University:

Mr. James Harvey Edmonston, The George Washington University;
Mr. Daniel Bernard Fisher, The George Washington University;
Mr. Solomon Kullback, The George Washington University;
Dr. Raymond J. Seeger, The George Washington University.

As Nominee of Yale University:

Mr. Augustus Henry Fox, Yale University.

At the meeting of the Council held on April 8, it was decided to omit the Christmas meeting in Chicago for 1932. The dates

for the 1933 spring meeting in Chicago were set for April 14–15, 1933.

On Friday afternoon, Professor R. L. Wilder delivered a Symposium Lecture on *Point sets in three and higher dimensions, and their investigation by means of a unified analysis situs.*

At the beginning of the Saturday morning session, by invitation of the Program Committee, Professor E. P. Lane addressed the Society on *Surfaces and curvilinear congruences.*

On Friday evening members and their guests attended a dinner in the Hotel Windermere West. Professor D. R. Curtiss was toastmaster. Professor W. A. Manning extended a warm invitation to all those present to attend the summer meeting in Los Angeles. Professor R. D. Carmichael spoke of the Transactions. Professor H. L. Rietz paid tribute to the memory of Professor J. W. Young, and expressed the sorrow of all of Professor Young's friends and associates upon his death. One hundred nine people attended this dinner.

The meetings were held in Eckhart Hall. On Friday afternoon tea was served in the Commons Room, and throughout the meeting this room was available for social purposes.

Vice-President E. W. Chittenden and Professor L. M. Graves presided at the Friday morning session, Vice-President W. A. Manning Friday afternoon, and Professors A. B. Coble and E. P. Lane on Saturday morning.

Mr. Nathan was introduced by Professor I. A. Barnett, Dr. Robinson by Professor Chittenden, Miss Schottenfels by Professor Lane, Mr. Matison, Mr. Martin, and Mr. Botsford by Professor A. D. Michal, and Professor Hasse by Professor Albert. Although Professor Stouffer was not present, the work in his paper was briefly discussed by Professor Lane.

General Session, Friday morning.

1. *Functional invariants of integro-differential equations.* Preliminary report, by Professor I. A. Barnett. (Abstract No. 38 5–127.)

2. *On the invariant character of a set of differential equations*, by Mr. Gordon Fuller. (Abstract No. 38–5–128.)

3. *On approximately conformal maps of surfaces.* Preliminary report, by Professor Tibor Radó. (Abstract No. 38–5–129.)

4. *On the Padé table for a power series having a corresponding continued fraction in which the coefficients have limiting values*, by Professor H. S. Wall. (Abstract No. 38–3–75.)

5. *Sphere geometry and the conformal group in function space*, by Mr. D. S. Nathan. (Abstract No. 38–5–130.)

6. *Sampling from many parent populations*, by Professor W. D. Baten. (Abstract No. 38–3–74.)

7. *Some properties of normally correlated variables*, by Dr. A. T. Craig. (Abstract No. 38–5–131.)

8. *Certain cardinal numbers associated with a point in a general topological space*, by Dr. Selby Robinson. (Abstract No. 38–5–132.)

9. *On the interchange of limit and integral for classes of functions in real, complex, and quaternionic numbers*, by Miss I. M. Schottenfels. (Abstract No. 38–5–133.)

10. *Sufficient conditions for a problem of Mayer in the calculus of variations*, by Professor G. A. Bliss and Dr. M. R. Hestenes. (Abstract No. 38–5–134.)

11. *Sufficient conditions for a problem of Mayer with variable end-points*, by Dr. M. R. Hestenes. (Abstract No. 38–5–135.)

12. *A new method for finding the numerical sum of an infinite series*, by Professor C. C. Camp. (Abstract No. 38–3–77–*t*.)

13. *Summation factors for double series*, by Professor W. H. Durfee. (Abstract No. 38–5–136–*t*.)

14. *On the nature of the solutions of certain types of integro-differential equations*, by Mr. Harry Matison. (Abstract No. 38–5–137–*t*.)

15. *Contribution to the theory of analytic functions in abstract spaces*. Paper II, by Mr. R. S. Martin. (Abstract No. 38–5–138–*t*.)

16. *On the trigonometric developments of certain doubly periodic functions of the second kind*. by Professor M. A. Basoco. (Abstract No. 38–5–139–*t*.)

General Session, Saturday morning.

17. *Surfaces and curvilinear congruences*, by Professor E. P. Lane. (Abstract No. 38–3–70.)

18. *A geometrical determination of the canonical quadric of Wilczynski*, by Professor E. B. Stouffer. (Abstract No. 38–3–67–*t*.)

19. *A classification of quadrics in affine n-space by means of arithmetic invariants*, by Dr. R. S. Burington. (Abstract No. 38–3–72.)

20. *Concerning the theory of systems of linear inequalities*, by Dr. I. J. Schoenberg. (Abstract No. 38–5–140.)

21. *On primitive groups containing circular permutations of degree p^α or $2p^\alpha$*, by Professor W. A. Manning. (Abstract No. 38–5–141.)

22. *A construction of non-cyclic normal division algebras*, by Professor A. A. Albert. (Abstract No. 38–3–73.)

23. *Linear associative algebras of infinite order whose elements satisfy finite algebraic equations*, by Professor H. H. Conwell. (Abstract No. 37–11–402.)

24. *Quadratic cycles*, by Professor J. S. Turner. (Abstract No. 38–5–142.)

25. *Subvarieties of a field*, by Mr. C. R. Worth. (Abstract No. 38–5–143.)

26. *Analytic theory of singular difference equations*, by Professor G. D. Birkhoff and Dr. W. J. Trjitzinsky. (Abstract No. 38–5–144.)

27. *Note on the solution of normal systems of linear differential equations with constant coefficients*, by Dr. Walter Bartky and Professor R. W. Barnard. (Abstract No. 38–5–145.)

28. *On the second variation of the area-integral*, by Professor Tibor Radó. (Abstract No. 38–5–146.)

29. *Sketch of an important solvable problem in formal modular covariants*. Preliminary report, by Professor O. E. Glenn. (Abstract No. 38–5–147–*t*.)

30. *On the contact of two space curves*, by Professor E. B. Stouffer. (Abstract No. 38–3–71–*t*.)

31. *The new Einstein geometry and some of its extensions*, by Professor A. D. Michal and Mr. J. L. Botsford. (Abstract No. 38–5–148–*t*.)

32. *A determination of all normal division algebras over an*

algebraic number field, by Professor A. A. Albert and Professor Helmut Hasse. (Abstract No. 38–3–68–*t*.)

33. *On normal simple algebras*, by Professor A. A. Albert. (Abstract No. 38–3–69–*t*.)

34. *Canonical binary trilinear forms*, by Mr. Rufus Oldenburger. (Abstract No. 38–3–76–*t*.)

35. *Further generalizations of certain classes of extreme numbers relative to certain symmetric equations in n reciprocals*, by Professor H. A. Simmons. (Abstract No. 38–5–149–*t*.)

36. *Non-absolutely convergent integrals with respect to a function of bounded variation*, by Professor R. L. Jeffery. (Abstract No. 38–5–150–*t*.)

37. *The variable end point problem of the calculus of variations including a generalization of the classical Jacobi conditions*, by Dr. A. E. Currier. (Abstract No. 38–5–151–*t*.)

M. H. INGRAHAM
Associate Secretary

TWO BOOKS ON DIFFERENTIAL GEOMETRY

Eléments de Géométrie Infinitésimale. By Gaston Julia. Paris, Gauthier-Villars, 1927. vi+242 pp.

Lehrbuch der Differentialgeometrie. By A. Duschek and W. Mayer. Band I, *Kurven und Flächen im Euklidischen Raum*, von A. Duschek. vi+250 pp. Band II, *Riemannsche Geometrie*, von W. Mayer. vi+245 pp. Leipzig and Berlin, B. G. Teubner, 1930.

It is interesting to compare Julia's book with the first volume of Duschek-Mayer's work, as they deal with the same subject, classical differential geometry. Both books use invariant notation; both endeavor to be accurate in the formulation of the theorems; both are of about one size. Still they differ greatly in general aspect and in material.

Julia's book follows more the classical pattern, and refers often to the treatises of Picard, Goursat, de la Vallée Poussin, Darboux, and to the work of Humbert. It uses vector methods for the formulation of certain general theorems, but often slips into coordinate notation, as required in special problems.

Duschek's book, on the contrary, persistently attempts to use not only vector methods, but also tensor calculus in ordinary differential geometry. As such, it is a pioneer work, with the possible exception of Ricci's never printed *Lezioni sulla Teoria delle Superficie* and J. E. Campbell's *Course of Differential Geometry* (1926), which, however, differ very much from Duschek's treatise. This results in a tendency to dwell upon theorems of a general nature.

A striking difference lies in the large amount of space that Julia devotes to the theory of contact and to the theory of envelopes (pp. 9–72, one-fourth of the book). Duschek devotes to this subject only a short discussion. Is it because Julia is in first instance an analyst? The modern theory of contact and envelopes was indeed introduced into differential geometry by two analysts, Lagrange and Cauchy. This fact makes Julia's book one of the best sources of information on contact and on envelopes. We find here contact of plane curves, of space curves, of curves and surfaces, and of surfaces. Then we have the discussion of the envelopes of systems of

(a) plane curves of equation $f(x, y, \alpha)=0$;
(b) plane curves of equation $f(x, y, \alpha, \beta)=0$, $\phi(\alpha, \beta)=0$;
(c) surfaces of equation $f(x, y, z, \alpha)=0$;
(d) surfaces of equation $f(x, y, z, \alpha, \beta)=0$, $\phi(\alpha, \beta)=0$;
(e) surfaces of equation $f(x, y, z, \alpha, \beta)=0$;
(f) space curves of equation $f(x, y, z, \alpha)=0$, $g(x, y, z, \alpha)=0$;
(g) space curves of equation $f(x, y, z, \alpha, \beta)=0$, $g(x, y, z, \alpha, \beta)=0$;

which discussion leads to the theory of congruences of curves, with mention of focal properties.

We should have liked to see here a more detailed discussion of the behavior of the envelope, taking higher derivatives into account. Apart from the paper by Risley and McDonald in the Annals of Mathematics of 1910–11 (second

series, vol. 12), and a paper by de la Vallée Poussin, reprinted in the sixth edition of his *Cours d'Analyse Infinitésimal* (Tome II, 1928, published after Julia's book), very little has been done on this important subject.

In later parts of the book Julia remains interested in the subject of congruences. He studies line congruences, especially normal congruences, with some interesting examples. Ruled surfaces and developable surfaces are discussed.

Other points treated in Julia's book are the classical theory of space curves and surfaces, Bour's theorem, and conformal representation applied to map projection. In an appendix we find remarks on imaginary elements and on vector notation. Vector notation is, in this treatise, just a short way to write three component equations in euclidian three-space. It is not used for curvilinear coordinates on the surface.

Duschek's book, on the contrary, has the use of Ricci notation as its special feature. The selection of the material is in accordance with this notation. In an introduction he explains the principal group conceptions and the Erlanger program. Vector notation is replaced by component notation v_i, $i=1, 2, 3$, in accordance with the dialectic method of Ricci calculus, which brings vector calculus to a higher perfection by the consistent use of components. Scalar products are written $v_i w_i$, vector products are obtained with the aid of the unit trivector e_{ijk}, so that

$$e_{ijk}v_i w_k = p_i$$

stands for

$$v \times w = p$$

in the Gibbs notation.

The elementary theory of curves can easily be translated into such notation. For the surface a new set of vectors and tensors is necessary, those defined with respect to transformations of curvilinear coordinates on the surface. Here we get vectors v_α, $\alpha=1, 2$, and tensors $p_{\alpha\beta}$, $\alpha, \beta=1, 2$. Latin indices run from 1 to 3, Greek indices from 1 to 2. Now we can develop readily the general properties of curves on the surface. We also find a discussion of analytic curves with complex variables, of so-called ametric parameters on analytic surfaces in the complex domain (these are parameters for which $ds^2=g_{12}du_1du_2$), of the determination of a surface by its first and second differential form, of Gauss-Bonnet's theorem, of geodesic conic sections, and of Liouville surfaces. As an example of differential geometry in the large we find the theorem on the impossibility of deforming a closed ovaloid.

Julia and Duschek differ in the way they introduce the positive direction of the principal normal to a space curve. For Julia this direction is that from the point of the curve to the center of curvature. The radius of curvature is always positive. Duschek, as does Blaschke, leaves the direction indeterminate, the curvature k and the unit normal vector n being only related by the formula $dt/ds=kn$, t representing the unit tangent vector. Julia's method leads to a difficulty in the case of plane curves, where a point of inflexion suddenly turns the moving "two-leg" through an angle of 180°. He therefore changes the definition for the plane. Duschek's definition does not lead to this, and it can be shown that in the plane the sign of k can be uniquely defined.

In Duschek's book there is no thorough discussion of the relations between tensors on the surface and tensors in space. The "induction" of a tensor in a surface by a tensor in space is not introduced. This is, however, regularly done in the second volume of Duschek-Mayer's treatise. In the first volume we therefore find the second fundamental tensor $b_{\alpha\beta}$ defined in this way:

$$b_{\alpha\beta} = \frac{\partial^2 x_i}{\partial u_\alpha \partial u_\beta} \nu^i, \qquad (i = 1, 2, 3; \alpha, \beta = 1, 2),$$

where ν^i are the components of the surface normal and $x_i = x_i(u_\alpha)$ are the equations of the surface. This is sufficient for ordinary differential geometry.

Covariant differentiation on the surface is introduced after the discussion of space curves and the elementary theory of surfaces. It is mainly used for a renewed discussion of the Gaussian curvature tensor, for the proof that a surface is determined in shape by its two fundamental tensors, for infinitesimal deformation, and for geodesics.

A chapter on some special subjects (ruled surfaces, minimal surfaces) concludes this interesting book.

Walther Mayer has written the second volume of Duschek-Mayer's textbook on differential geometry, and it has in many respects a character similar to that of the first volume. It deals with Riemannian geometry, and is a very valuable addition to the literature on this subject. It contains a careful introduction to tensor calculus, without which the study of Riemannian manifolds seems to be very difficult. Then follow a chapter on curves in euclidean n-space with the Frenet formulas for Riemannian manifolds, and a forty-page introduction to the calculus of variations with application to geodesics. After this we get the parallel displacement of Levi-Civita, geodesic manifolds, and a chapter on manifolds immersed in a Riemannian manifold. The spaces of constant Riemannian curvature receive special attention. A special feature of the book is Chapter IX, containing what the author calls "Das Formenproblem." Here an old question is finally settled, namely the generalization to n dimensions of Bonnet's theorem that a surface is determined in shape by its first and second fundamental tensors. Here the complete theory is given for an l-dimensional manifold ($l < n$), in an n-dimensional euclidean space. The formulas constructed for this purpose are the generalization of the Frenet formulas to this general case. Since that time Schouten and van Kampen have restated the results in another way (Mathematische Annalen, vol. 105 (1931); see the remark by Duschek in Zentralblatt für Mathematik, 26 Oktober 1931, p. 153).

An appendix gives a generalization of Meusnier's theorem, the integral theorem of Gauss in Riemannian manifolds, and the derivation of the Eulerian equations for the gyroscope in tensor calculus, where they appear as an almost trivial result of the fundamental equations.

As a result of the collaboration of the two Viennese scholars we therefore have an extremely interesting addition to our textbooks on differential geometry.

D. J. STRUIK

VAN DER WAERDEN ON ALGEBRA

Moderne Algebra. By B. L. van der Waerden. Unter Benutzung von Vorlesungen von E. Artin und E. Noether. Berlin, Springer, 1931. Vol. I. viii+243 pp. Vol. II. vii+216 pp.

It will immediately occur to every reader of van der Waerden's new book that modern algebra is a subject quite different from the classical algebra built up in its last golden era by Dedekind, Weber, Frobenius, and Kronecker. It is true that a closer study often reveals that the main difference lies in the form of presentation, but it is equally true that in many instances the problems of modern algebra are broader and of a different character.

The new school of abstract algebra has developed into one of the strongest branches of present day mathematics in Germany. Its fundamental principles are closely related to Hilbert's ideas of a formal foundation of mathematics, reducing all theories to an axiomatic basis consisting of relational properties of undefined elements. It is of course nothing new to build up a mathematical theory from its axioms; the main problem of abstract algebra is however the determination of *all* systems with a given operational basis, i.e. to find the structural properties of all such systems. It is interesting to observe to what a remarkable extent this has been possible. An immediate consequence is an intimate knowledge of the fundamental assumptions of each theory and theorem; but still more important is the abstract identification of many mathematical investigations, also outside of algebra proper, which makes abstract algebra a unifying principle sorely needed in these times of specialization.

One of the central papers in abstract algebra is the well known analysis by Steinitz of the structure of fields. It has been followed by a vast number of investigations on the structure of groups, rings, ideals, hypercomplex systems, etc., works associated with the names of Artin, Krull, E. Noether, and others. For hypercomplex systems the investigations of Wedderburn and Dickson are outstanding.

This new *Algebra* proposes to be a guide to these investigations, and in many ways it is more than that; van der Waerden has coordinated the various investigations and he has tried as far as possible to consider them from the most general point of view. His book therefore gives considerably more than a summary of the previous theory, and it will certainly keep a prominent place among the books on algebra for many years to come. It is not a text book in the ordinary sense, and it requires a wide preliminary knowledge on the part of the reader, even though every subject is worked up from its foundations. But I am certain that advanced students and mathematicians interested in algebra will study it with great pleasure.

The book is clearly written, well balanced in its content, and there is much to be praised and little to be criticised. In places the desire for great generality has made it a little obscure; the problems also seem somewhat too simple and I should have liked to have a more complete list of references. Due to the large quantity of facts which had to be crowded into a short book, one sometimes

feels that one would have preferred to have a little more of every subject, but this is after all only the earmark of a good book.

The first volume is on the whole more elementary than the second, and is in itself a good introduction to the abstract theory of fields, groups, and ideals. It starts with a discussion of numbers, sets, and the foundations of group theory. The next chapter contains the fundamental properties of rings and fields, isomorphisms, properties of polynomials in a ring, the principal concepts of the abstract ideal theory, and a discussion of the decomposition into prime elements. After constructing a quotient field for a commutative ring, van der Waerden observes that the corresponding problem for non-commutative rings is still unsolved; in a recent paper I have shown that in the case of a non-commutative ring without divisors of zero, the set of all formal quotients will only form a field in case the ring has the property that for any two given elements a and b one can find two others $a_1 \neq 0$, $b_1 \neq 0$ such that $a_1 a = b_1 b$. There are, however, as I have indicated by an example, other methods of enlarging a ring to a field, but it seems very hard to find a general principle for such a construction.

In the fourth chapter one finds the theory of polynomials and symmetric functions in a ring R or a field F. The decomposition into prime polynomials is treated at some length; among other results one will find the following: If the elements of the ring R are factorable in a finite number of steps, and R has only a finite number of units, then all polynomials with coefficients in R are factorable in a finite number of steps. The proof follows by a method of Kronecker. I should like to point out that one can give a necessary and sufficient condition from which the criterion of van der Waerden immediately follows, viz: The factors of polynomials with coefficients in a field F can be found in a finite number of steps, if the first degree factors can be determined in this way. It is only necessary to consider the case where the polynomial has different roots; if it has a factor of mth degree, one constructs the equation in F satisfied by any symmetric function of m of the roots; the possible roots in F of this equation give the possible coefficients of a factor.

The long Chapter 5 gives the abstract theory of fields in a somewhat simpler form than does Steinitz; it should be observed that van der Waerden calls an irreducible polynomial *separable* if all of its roots are different, *inseparable* if this is not the case; these terms are more suggestive than the *first* and *second kind* of Steinitz. In Chapter 7 the author returns to group theory and also treats the groups with operators studied by Krull and Noether, and then follow the applications to the general Galois theory of (finite) algebraic extensions. The fundamental correspondence-theorem for subgroups and subfields holds for all separable extensions and only for these. It is to be noted, however, that when the characteristic of the field is not zero, the Galois criterion for solvability by radicals does not apply to all cases.

In the following chapter one finds the proof of the existence of algebraically closed fields and the invariance of the degree of transcendency; these parts of the theory were rather complicated in the original treatment by Steinitz, but van der Waerden has succeeded in giving a clear-cut presentation. The last chapter contains Artin and Schreier's theory of abstract real fields and also a short paragraph on absolute values (Bewertungen). It would have been inter-

esting to have a more complete discussion of the latter theory, which contains many interesting results, but has never been presented in any textbook.

The second volume deals mainly with general ideal theory and non-commutative algebra. It would be tempting to discuss at some length the several new subjects which are here treated for the first time in a textbook, but I shall have to limit myself to a short description of the main character of the chapters.

Chapter 11 contains the general elimination theory of functions of several variables. The existence of a system of resultants is demonstrated, and used to prove the fundamental theorem of Hilbert. As an application the theorem of Bezout is proved for an arbitrary number of variables. Chapter 12 contains commutative ideal theory in domains in which E. Noether's chain condition holds; the principal result is the theorem by Lasker on the representation of an ideal as the greatest common factor of primary components. In Chapter 13 this theory is applied to the theory of polynomial ideals, which is virtually algebraic geometry in abstract form. One finds among other topics a rather complete discussion of the fundamental theorem of M. Noether, deduced from the theorem of Lasker. This abstract algebraic geometry has lately been one of the favorite subjects of the author, and I should have liked to see some of his further investigations included, for instance, on the "Abzählungsprinzip" of Zeuthen.

Chapter 14 contains investigations of Noether and Krull on the ideal theory in maximal integral (ganzabgeschlossenen) rings; this theory includes the foundation of the ordinary ideal theory in algebraic fields and it is directly inspired by the theory of Dedekind. Chapter 15 deals with linear systems, moduli, matrices, and elementary divisors for commutative and non-commutative domains. One finds the main properties of the generalized Abelian groups studied by Krull, and at the end a paragraph on quadratic and Hermitian forms. Chapter 16 contains the principal theorems on the structure of hypercomplex systems; the presentation is short, too short I am afraid, for a reader who is not already somewhat familiar with the subject. The final chapter contains the application of this theory to the representation of groups and the theory of characters of finite groups.

OYSTEIN ORE

SHORTER NOTICES

Das mathematische Werkzeug des Chemikers, Biologen, Statistikers, und Soziologen. By R. Fueter. Zweite verbesserte und vermehrte Auflage. Zürich and Leipzig, Füssli, 1930. 303 pp.

This book is addressed particularly to students of chemistry, biology, and medicine, with the idea of instructing them in the fundamental concepts of mathematics and the possibility of applications in the natural sciences. It is distinctly not a collection of formulas. The topics covered are differential and integral calculus, statistics, and a brief introduction to differential equations.

W. R. LONGLEY

Die Raum-Zeit-Philosophie des 19. Jahrhunderts. By Werner Gent. Bonn, Friedrich Cohen, 1930. xii+397 pp. Price 16 RM.

This volume and the author's earlier work on *Die Philosophie des Raumes und der Zeit* (1926) together give a systematic account of the history of philosophical speculation on space and time from Aristotle to the present day. There seems to be no other work which treats this subject comprehensively. The years 1768–69 mark the dividing line between the two volumes; this is the time at which Kant closed his precritical stage. The later volume is written so that it may be read independently of the earlier; nearly one fourth of it is given to Kant. The treatment of the nineteenth century appears to be quite adequate except for the purposed omission of the investigation of space and time in experimental psychology. A brief chapter is given to the problem of space and time in mathematical physics; but even here there is but little more than an allusion to the mathematical questions involved. The book's chief appeal must lie in its interest to philosophers.

R. D. CARMICHAEL

Komplexe Reihen. By H. Falckenberg. Berlin and Leipzig, de Gruyter, 1931. (Sammlung Göschen, No. 1027.) 140 pp.

This little book is complementary to the author's earlier text entitled *Elementare Reihenlehre* (Sammlung Göschen, No. 943). The first part, consisting of 28 pages, contains an elementary treatment of infinite series with complex terms, a topic omitted from the earlier book because of lack of space. The second part contains a collection of more than 100 problems arranged in groups by topics, with references to the corresponding text in both books. Many of the problems are routine exercises while others contribute appreciably to the theory. In the last group an elementary introduction to the theory of Dirichlet and Lambert series is given through the medium of problems. The third part of the book contains solutions of the problems given in the second part.

This volume provides a valuable convenience to the instructor who wishes problems for class use and it affords excellent material to the student who wishes to test his grasp of the subject matter and to cultivate skill in technique.

W. R. LONGLEY

Numerical Mathematical Analysis. By J. B. Scarborough. Baltimore, The Johns Hopkins Press, 1930. xiv+416 pp.

Mathematics has been defined as the science of avoiding computation, and in many fields of pure mathematics the ideal is to achieve results by reasoning which involves no computation. In recent years numerical mathematics has grown rapidly in connection with statistics, biology, and other sciences, and computation is a necessary consequence. It is in the fields of applied mathematics where numerical results are demanded that it becomes most desirable to avoid unnecessary and inaccurate computation. The object of Professor Scarborough's book "is to set forth in systematic manner and as clearly as possible the most important principles, methods, and processes used for obtaining numerical results; and also methods and means for estimating the accuracy of such results."

The book is not a treatise on computation in the sense of convenient forms and efficient arrangements. These are usually highly specialized and the fundamental principles with which the book deals are not so specialized. Notations peculiar to certain subjects have been avoided and the treatment has been made as elementary as is consistent with soundness. Assuming only a knowledge of elementary calculus, the author has succeeded in giving an excellent presentation of his material. Specialists in some of the topics may disagree with the choice of material and may regret the omission of a few methods like that of divided differences in the standard formulas of interpolation, but the author has made his choice with a definite view in mind. He has "tried everywhere to clear up the difficulties before the student meets them, so that no teacher or other source of information will be needed."

This ideal in the matter of explanation is particularly laudable in a work of this character because its greatest usefulness will be as a reference book when one needs to use methods which are not very familiar to him. After an exposition of the theory underlying a formula, the reader is shown how to use it and its limitations are carefully pointed out. The book would serve well as a classroom text. At the present time no large number of students pursue such a course, but an increasing number of mathematicians and scientists are performing work for which the methods here treated are essential. Since it seems likely that the book will be most widely used for individual study without the guidance of a teacher, its value would be increased if answers were given to most of the problems.

The following list of chapter headings will indicate the contents. 1. The accuracy of approximate calculations. 2. Interpolation (Newton's formulas). 3. Interpolation (central-difference formulas). 4. Interpolation (Lagrange's formula. Inverse interpolation). 5. The accuracy of interpolation formulas. 6. Interpolation with two independent variables. Trigonometric interpolation. 7. Numerical differentiation and integration. 8. The accuracy of quadrature formulas. 9. The solution of numerical algebraic and transcendental equations. 10. Graeffe's root-squaring method for solving algebraic equations. 11. The numerical solution of differential equations (method of successive approximations). 12. Convergence and accuracy of the iteration process. 13. Other methods for the numerical solution of differential equations (J. C. Adams, Runge-

Rutta, Milne). 14. The normal law of error and the principle of least squares. 15. The precision of measurements. 16. Empirical formulas. 17. Harmonic analysis of empirical functions.

Problems are given at the end of every chapter and a table of values of the probability integral is given in an appendix. Of particular importance for a work of reference is the adequate index.

W. R. LONGLEY

The Mathematical Part of Elementary Statistics. By Burton Howard Camp. New York, D. C. Heath and Company, 1931. 409 pp.

Professor Camp's new book is a most thoroughly worked out and comprehensive textbook for teaching the student with the minimum of mathematical preparation the maximum about the mathematical tools of statistical analysis. In fact the extent to which mathematical formulas and results which depend on mathematical considerations of a more or less advanced character for their complete understanding are explained and made available to students whose mathematical equipment is not supposed to go beyond analytic geometry, almost makes this book a new departure in its class.

The book is divided into three parts, the third part being tables with an introduction to them. The first part covers a minimum course for one semester and avoids the more difficult notions of Part II. Part II takes up the point binomial, the Gram-Charlier series, sampling, the Tchebycheff inequality and modifications of it, the X-square test, and a more thorough discussion of correlation including an admirable geometrical explanation of multiple correlation. There is also an introductory chapter on finite differences. A feature of the tables is their adaptability to the problem of easily getting a good approximation to the sum of a group of consecutive terms of a skew point binomial.

Most texts on statistics are sadly lacking in problems; every teacher of statistics knows how hard they are to get up, and Professor Camp is to be congratulated on the full and excellent tests he has provided.

Because of the author's wide first-hand knowledge of the subject, what he has to say about mathematical statistics is accurate. (It seems at least doubtful to the reviewer whether the logical difficulties involved in any attempt to define probability should be mentioned in a book of this character. The author does not mention them.) But the reviewer cannot feel entirely comfortable about teaching so much more about the subject than can be taught of it. He is thoroughly convinced of the value of the study of mathematical statistics and probability as a discipline, of its value for its own sake as a branch of mathematics. And he confesses to being dubious about the practical utility of the knowledge one can gain at second-hand about matters as complex as many of those dealt with in the second half of Professor Camp's book. Perhaps it should be stated explicitly here that this review is written from the point of view of one interested in mathematical statistics for its own sake. But there is great pressure from outside upon mathematics departments to teach the practical methods of mathematical statistics to students of small mathematical preparation and maturity, and wherever this is undertaken Professor Camp's book will be a very valuable aid.

C. C. CRAIG

A Source Book in Mathematics. Edited by David Eugene Smith. New York, McGraw-Hill, 1929. xvii+701 pp. Price $5.00.

A grant of $10,000 by the Carnegie Corporation to the American Philosophical Association in 1927 has made possible a series of source books in the history of the sciences. The first volume issued deals appropriately with astronomy and the second volume, issued in 1929, deals with mathematical science.

The general editor of the series is Professor Gregory D. Walcott. For the volume on mathematics the editorship was entrusted naturally to David Eugene Smith whose activity in the history of mathematics has given America a real position in this field. An advisory committee consisting of R. C. Archibald of Brown, Florian Cajori (deceased), and L. E. Dickson, gave further guarantee of a work of high order of scholarship.

This series of some ninety-six excerpts from classical contributions to the development of mathematics marks a notable step in the progress of the history of science in America. For many years courses in the history of mathematics have been given in most institutions without the possibility of reference to original sources. Even in those institutions where many of the original works are available the language difficulty has proved a barrier to any wide use of the original materials. With this volume the English-speaking student of mathematics has the possibility of easy access to many of the most fundamental contributions to mathematics.

As the plan of the series of which this is a part does not include the development of mathematics before the invention of printing, many fundamental developments of elementary mathematics are left untouched. Every student of the history of mathematics will second the wish expressed by the editor that a volume devoted to the earlier phases of mathematics may be included in the series.

The various mathematical topics are arranged under five divisions: The Field of Number, The Field of Algebra, The Field of Geometry, The Field of Probability, The Field of Calculus, Functions, Quaternions.

It is particularly worthy of note that so much material has been included which is of immediate interest even to high school students. Robert Recorde's *Declaration of the Profit of Arithmeticke* is material which is of interest to any cultured person. The work on decimal fractions, irrational numbers, complex numbers, π, logarithms, the slide rule, the cubic and biquadratic, Horner's method, the law of sines and other articles on trigonometry, all connect directly with material offered today in America in the larger high schools.

Ninety-six topics are not sufficient to cover the really important developments of modern mathematics and doubtless in some later edition other topics and other classical names will be included. However the editor and the committee have rendered a valuable service to mathematical science, emphasizing the international character of the developments in mathematics and the essential underlying unity of the contributions which have been made over a period of four hundred and fifty years.

The printing has been done with a great deal of care. The historical notes have also been well edited. Doubtless the reference to the trigonometric work by Regiomontanus (p. 432) should not refer to it as the first work devoted solely

to trigonometry, since an Arab, Nasir ed-din al-Tusi (1201–1274), wrote a separate treatise on the subject. Such errors are more or less unavoidable. Particular attention is directed to this point, however, since it would seem that special effort should be made to give to the Arabs, the Hindus, the Egyptians, and Babylonians, recognition of their achievements in mathematics, since this emphasizes the universality of the appeal of mathematics.

This handsome volume should find a place in every high school and university library. Through its pages generations of students will find inspiring contact with the old masters of mathematics whose work has made possible so many of the most remarkable developments of modern civilization.

L. C. KARPINSKI

Theorie der Raumkurven und krummen Flächen. By V. Kommerell and K. Kommerell. I. *Krummung der Raumkurven und Flächen.* 205 pp. II. *Kurven auf Flächen. Specielle Flächen. Theorie der Strahlensystem.* 194 pp. (Göschen's Lehrbücherei. I. Gruppe: Reine und angewandte Mathematik. Band 20 und 21.) Berlin, de Gruyter, 1931.

These two volumes are a fourth edition, completely revised, of three volumes of the Sammlung Schubert of which the latest editions were published in 1921. The chief differences from the earlier editions, as pointed out in the preface, are: the introduction from the beginning of the parametric representation of surfaces; greater rigor of presentation; the introduction of Levi-Civita's parallel displacement and the Riemann-Christoffel curvature tensor with a brief reference to relativity; a new treatment of the Gauss-Bonnet formula; a fundamental revision of the treatment of the transformation of parameters and of differential parameters. K. Kommerell is chiefly responsible for the scientific content, V. Kommerell for the didactic side of the work.

These volumes seem to us to give a full and extremely good presentation of classical differential geometry. We agree with the statement made by the authors in the preface that the diligent student of this book will be in a position to read original papers and even himself to undertake scientific investigation. We think that the book is not altogether easy to read and hesitate to recommend it to a beginner in the subject. As to content it is roughly comparable to Eisenhart's *Differential Geometry*. The treatment follows classical methods with occasional use of vector calculus, the latter, we think, not used with any gain in simplicity and not enough used to give the reader any familiarity with the subject. The work is rich in reference to original sources and contains several interesting historical discussions, notably those of minimal surfaces and of non-euclidean geometry. The last forty pages of text give an excellent treatment of rectilinear congruences. At the end of the second volume there are 115 exercises covering the whole work. These vary greatly in difficulty. For a number of them references to the original discussions of these problems are given. Each volume has a table of contents at the beginning and, at the end, an index of authors referred to and an index of topics. The book seems remarkably free from errors: we have noticed but one, a misplaced "2" in the equation of the catenary on page 76 of volume II.

J. K. WHITTEMORE

Leçons Élémentaires sur le Calcul Numérique. By E. Cahen and C. Michel. Paris, Gauthier-Villars, 1931. 111 pp.

In this book the authors emphasize the laborious feature of mathematics which is almost always avoided by students and teachers in American colleges. For the most part, the problems treated are those which arise in a first course in the calculus and the purpose is to obtain numerical results to a rather high degree of accuracy. Numerical equations, both algebraic and transcendental, are solved by Newton's method and by the method of successive approximations. Some of the problems demand results to 12 decimal places. Polynomials are evaluated by synthetic division and by interpolation and extrapolation, using all of the successive differences. Considerable attention is given to the sum of a series, when the degree of accuracy is prescribed in advance. The last chapter is devoted to the evaluation of definite integrals by means of series, the trapezoidal rule, and Simpson's rule.

W. R. LONGLEY

Die Pseudosphäre und die Nichteuklidische Geometrie. By F. Schilling. Leipzig, Teubner, 1931. v+70 pp.

In the second volume of his *Projektive und Nichteuklidische Geometrie*, which I reviewed in this Bulletin a short time ago, Professor Schilling gives a comprehensive treatment of non-euclidean geometry from the projective standpoint. The present monograph aims at a more popular presentation of hyperbolic geometry by models of the pseudosphere and its map on the upper half of a complex plane, or Cartesian plane in which the lines are represented by semicircles cutting the real x-axis orthogonally. The analytic geometry and trigonometry of hyperbolic geometry are worked out on this basis. This method affords undoubtedly the shortest and most effective approach to the subject, and Schilling is thus very successful in presenting and elucidating a very interesting geometric field to persons with only elementary mathematical training.

ARNOLD EMCH

Caratteristiche dei Sistemi Differenziali e Propagazione Ondosa. By Tullio Levi-Civita. Lezioni raccolte dal Dott. G. Lampariello. Bologna, Nicola Zanichelli, 1931. vii+108 pp.

These lectures give a clear account of that branch of partial differential equations in which attention is focused on the problem of Cauchy and its exceptional cases. This problem leads naturally to the idea of characteristics and the relation of these to waves of discontinuity is elucidated by the formulation of kinematical and dynamical conditions of compatibility at the wave-front.

When the partial differential equation of the characteristics is of the first order and is expressed in a normal form by solving for the partial derivative with respect to the time, a theory of rays may be based on an associated set of differential equations of the Hamiltonian form which the authors call the bicharacteristics, thus adopting the terminology of Hadamard instead of that of Cauchy. The theory is well illustrated by examples from hydrodynamics, optics, and the recent theory of de Broglie waves.

H. BATEMAN

NOTES

The Spring Meeting of the American Association for the Advancement of Science will be held at the University of Syracuse during the week of June 20–25. Section A (Mathematics) will meet on Wednesday, June 22, with addresses by Professor H. M. Gehman on *Homeomorphic geometry of the projective plane*, by Professor W. A. Hurwitz on *Logical foundations for groups and fields*, and by Professor J. Shohat on *Interpolation*. Professor W. B. Carver will deliver a general address before the Association on *A layman's introduction to higher mathematics*, and Professor E. R. Hedrick will participate in a symposium on general educational problems.

A new Chinese scientific journal has recently been founded, the Science Reports of the National Tsing Hua University, Peiping, Series A, Mathematical and Physical Sciences. The first two numbers are in English.

The Academy of Sciences of the Ukraine, Kief, has begun publication of two new journals, the Bulletin de la Classe des Sciences Naturelles et Techniques and the Journal du Cycle Mathématique.

The Yale Law Journal for March, 1932, contains an article on *The nature of the doctrinal function and its rôle in rational thought*, by Professor C. J. Keyser, of Columbia University.

The Comité International d'Histoire des Sciences will hold its fourth annual meeting in Paris, May 13–16, 1932.

Professors W. H. Bussey and G. C. Evans have been elected vice-presidents of the Mathematical Association of America, and Professors H. E. Buchanan, W. B. Ford, E. R. Hedrick, and W. R. Longley members of its Board of Trustees.

The Belgian Academy has awarded its de Potter prizes, for the period 1928–1930, in astronomy and mathematics, respectively, to Professors Germay and Lepage.

The Scientific and Research Institute of Mathematics and Mechanics of the University of Moscow has awarded a prize of 750 roubles to L. S. Pontrjagin, member of that Institute, for his paper *Über stetige algebraische Körper*, recently published in the Annals of Mathematics, (2), vol. 33 (1932), pp. 163–174.

The British Institution of Electrical Engineers has awarded its Faraday medal to Sir Oliver Lodge.

Cambridge University has awarded Smith's prizes to D. W. Babbage, of Magdalene College, for an essay on *Cremona transformations*, and H. M. Taylor, of Clare College, for an essay on *The anomolous scattering of X-rays*, and Rayleigh prizes to J. Cossar, of Magdalene College, for an essay *On Fourier integrals*, and V. V. Narliker, noncollegiate, for an essay on *Cosmogony and astronomical dynamics*.

The gold medal of the Royal Astronomical Society has been awarded to Dr. R. G. Aitken, of the Lick Observatory, for his work on double stars.

Dr. Irving Langmuir, of the General Electric Company, has been awarded the prize of the Popular Science Monthly for notable scientific achievement.

Professor Henri Villat, of the Sorbonne, has been elected a member of the Paris Academy of Sciences in the section of mechanics, as successor to the late Gabriel Koenigs.

It will be of particular interest to those who know and honor him, and to those interested in correct principles of academic tenure, to learn that Senator Vito Volterra, professor of mathematical physics at the University of Rome since 1900, has been dismissed from active service for refusal to take the university oath recently required by the fascist government.

The Council of the Royal Society of London has recommended the following mathematicians for election as fellows of that Society: D. R. Hartree, Beyer professor of applied mathematics, University of Manchester, and H. W. Turnbull, professor of mathematics, University of St. Andrews.

Sir James Hopwood Jeans has been elected an honorary member of the Washington Academy of Sciences.

At a special convocation called to commemorate the bicentenary of the birth of Washington, George Washington University conferred honorary doctorates of science on Professors C. N. Moore, of the University of Cincinnati, and J. F. Ritt, of Columbia University, both alumni of the University.

The French government has conferred the designation *Officier d'Académie* and the decoration *palmes académiques* on Professor E. R. Hedrick, of the University of California at Los Angeles, "for services rendered to the cause of culture and of science." The ceremony was performed by the French Consul at Los Angeles, M. Henri Didot, on April 13, 1932.

Associate Professor F. S. C. Northrop, of Yale University, has been awarded a Guggenheim fellowship, "to review and evaluate current conceptions of the nature of mathematics and formal logic, and to determine the status of form in nature."

Professor Richard Baldus, of the Karlsruhe Technische Hochschule, has been appointed professor of geometry at the Munich Technische Hochschule.

Dr. G. Mammana, of the University of Cagliari, has been promoted to a professorship of the calculus.

Mr. R. H. Fowler, fellow of Trinity College, Cambridge, has been appointed to the John Humphrey Plummer professorship of mathematical physics at Cambridge University.

The title of fellow of University College, London, has been conferred upon Professor A. R. Richardson, of the University College of Swansea.

Dr. G. F. J. Temple, reader in mathematics at Imperial College, Royal College of Science, University of London, has been appointed to the chair of mathematics at King's College, in the same University.

Professor P. Debye, of the Physical Institute of Leipzig, has been appointed Scott lecturer at Cambridge University for the year 1932. Professor Debye has recently delivered a series of lectures on X-ray scattering and molecular structure at the Massachusetts Institute of Technology.

Dr. W. A. Shewhart, of the Bell Telephone Laboratories, will lecture at the University of London in the spring of 1932 on the rôle of statistical method in

industrial standardization, and will hold a series of conferences on theoretical and applied statistics in Great Britain and on the Continent. Dr. Shewhart has recently been elected a fellow of the Royal Statistical Society of England.

The following seventy-six doctorates with mathematics or mathematical physics as major subject were conferred during 1931 in universities in the United States and Canada; the major subject is mathematics unless otherwise specified. The university, month in which the degree was conferred, minor subject (other than mathematics), and title of dissertation are given in each case if available.

D. B. Ames, Yale, June, *A resonance problem in celestial mechanics.*

F. S. Beale, Michigan, *On the solutions of systems of linear difference equations with polynomial coefficients.*

H. F. Bohnenblust, Princeton, April, *The absolute convergence of Dirichlet series.*

Samuel Borofsky, Columbia, April, *Expansion of analytic functions into infinite products.*

Sister Leonarda Burke, Catholic, June, minors in education and physics, *On a case of the triangles in- and circumscribed to a rational quartic curve with a line of symmetry.*

L. E. Bush, Ohio State, August, *Some properties of algebras without moduli.*

S. S. Cairns, Harvard, June, *The cellular division and approximation of regular spreads.*

Helen Calkins, Cornell, November, *Some implicit functional theorems.*

A. B. Cardwell, Wisconsin, June, major in physics, minor in mathematical physics, *Photoelectric and thermionic properties of metals.*

Evelyn T. Carroll, Cornell, October, *Systems of involutorial birational transformations contained multiply in special linear line complexes.*

Emily M. Chandler (Mrs. H. H. Pixley), Chicago, August, *Waring's theorem for fourth powers.*

Max Coral, Chicago, August, *The Euler-Lagrange multiplier rule for double integrals.*

A. T. Craig, Iowa, June, *On the distribution of certain statistics derived from small random samples.*

J. H. Dillon, Wisconsin, June, major in physics, minor in mathematics, *Photoelectric properties of zinc single crystals.*

J. E. Donohue, Columbia, March, *Concerning the geometry of the second derivative of a polygenic function.*

H. L. Dorwart, Yale, June, *Certain types of criteria for the irreducibility of polynomials.*

R. D. Douglass, Massachusetts Institute of Technology, June, minor in physics, *Stirling expansions derived by means of finite de la Vallée-Poussin summation.*

Ben Dushnik, Michigan, *On the Stieltjes integral.*

J. N. Eastham, Catholic, June, minors in civil engineering and physics, *The triangles in- and circumscribed to the tacnodal rational quartic curve with residual crunode.*

H. C. T. Eggers, Michigan, *The graduation of experimental data.*

J. M. Feld, Columbia, November, *Projective pedal transformations and birational contact transformations.*

N. C. Fisk, Michigan, *An investigation of surfaces in euclidean four-space by means of three-vectors.*

A. L. Foster, Princeton, January, *A continuous development of formal logic in finite terms.*

Sister Mary de Lellis Gough, Catholic, June, minors in education and physics, *On the condition for the existence of triangles in- and circumscribed to certain types of the rational quartic curve and having a common side.*

Eli Gourin, Columbia, May, *On irreducible polynomials in several variables which become reducible when the variables are replaced by powers of themselves.*

M. L. Hartung, Wisconsin, June, minor in mathematical physics, *On a family of integral equations with discontinuous kernels.*

T. W. Hatcher, Cornell, August, minor in engineering, *Symmetric strain in an infinite plate with a circular hole.*

R. A. Hefner, Chicago, August, *The condition of Mayer for discontinuous solutions of the Lagrange problem.*

C. T. Holmes, Harvard, June, *The approximation of harmonic functions in three dimensions by harmonic polynomials.*

H. N. Hubbs, Cornell, February, minor in physics, *Rational quintic surfaces without double curves.*

W. R. Hutcherson, Cornell, June, minor in physics, *Maps of certain cyclic involutions on two dimensional carriers.*

D. R. Inglis, Michigan, *Atomic problems in the perturbation theory of quantum mechanics.*

Sister Mary Cordia Karl, Johns Hopkins, June, *The projective theory of orthopoles.*

Donald McDonough, Pennsylvania, June, *On the expansion of a certain type of determinant.*

W. O. Menge, Michigan, *A direct method of obtaining transformations to canonical forms.*

H. L. Miller, Cincinnati, June, *On the summability of non-integral orders of the double Fourier series.*

G. E. Moore, Illinois, June, minor in astronomy, *The four-termed theta identities arising from a generalization of the Weddle surface in* S_4.

C. B. Morrey, Harvard, June, *Invariant functions of conservative surface transformations.*

Sister Charles Mary Morrison, Catholic, June, minors in philosophy and physics, *The triangles in- and circumscribed to the biflecnodal rational quartic curve.*

M. E. Mullings, Cincinnati, June, minor in physics, *On Gibbs's phenomenon in the double Fourier series.*

C. V. Newsom, Michigan, *On the behavior of entire functions in distant portions of the plane.*

E. G. Olds, Pittsburgh, June, *The nature of the distributions in small samples.*

A. L. O'Toole, Michigan, *On symmetric functions and symmetric functions of symmetric functions.*

L. J. Paradiso, Cornell, October, minor in economics, *Solutions of bounded variation of the Fredholm Stieltjes integral equation.*

W. V. Parker, Brown, June, *The addition formulas for hyperelliptic functions.*

Jesse Pierce, Michigan, *Logarithmic solutions of systems of differential equations and solutions in the vicinity of isolated singular points.*

H. H. Pixley, Chicago, August, *A problem in the calculus of variations suggested by a problem in economics.*

Saul Pollock, California (Berkeley), May, *Graphic representation in enumerative geometry.*

J. E. Powell, Chicago, August, *Edge conditions for multiple integrals in the calculus of variations.*

H. R. Pyle, California (Berkeley), May, *The conditions for conformality in the elliptic and hyperbolic geometries.*

Mina S. Rees, Chicago, March, *Division algebras associated with an equation whose group has four generators.*

B. J. Roberts, Iowa, August, minor in physics, *On continua in vectorial spaces.*

H. A. Robinson, Johns Hopkins, March, *Case of planar motion in which one centrode and one path are circles.*

S. L. Robinson, Iowa, August, minors in physics and philosophy, *A study of simple and multiple transitivity in topological spaces.*

A. E. Ross, Chicago, August, *On representation of integers by indefinite ternary quadratic forms.*

T. A. Rouse, Wisconsin, June, major in physics, minor in mathematics, *Characteristic X-ray absorption.*

R. G. Sanger, Chicago, June, *Functions of lines and the calculus of variations.*

M. G. Scherberg, Minnesota, June, minor in physics, *The degree of convergence of a series of Bessel functions.*

Mabel F. Schmeiser, Ohio State, August, *Properties of arbitrary functions concerning approach to a straight line.*

C. Grace Shover, Ohio State, December, *On the class number and ideal multiplication in a rational linear associative algebra.*

James Singer, Princeton, April, *Three-dimensional manifolds and their Heegaard diagrams.*

P. K. Smith, Illinois, June, minor in physics, *On solutions of linear partial differential equations near singular points.*

T. L. Smith, Harvard, June, *The Birkhoff fluid theory of electricity.*

I. S. Sokolnikoff, Wisconsin, June, *On a solution of Laplace's equation with an application to the torsion problem for a polygon with reentrant angles.*

F. W. Sparks, Chicago, August, *Universal quadratic zero forms in four variables.*

R. C. Stephens, Iowa, August, minor in physics, *Continuous functions on abstract sets.*

W. T. Stratton, Washington (Seattle), August, minor in physics, *A study of general polar tangent curves.*

Mildred E. Taylor, Illinois, August, minor in physics, *A determination of the types of planar Cremona transformations with not more than 9 F-points.*

C. C. Torrance, Cornell, June, minor in physics, *On plane Cremona triadic characteristics.*

Sister Mary Felice Vaudreuil, Catholic, June, minors in education and physics, *Two correspondences determined by the tangents to a rational cuspidal quartic curve with a line of symmetry.*

C. C. Wagner, Michigan, *A statistical study of the dependence for four chance variables.*

W. G. Warnock, Illinois, May, minor in astronomy, *On the geometry of groups of line configurations.*

H. E. Wheeler, Chicago, June, *An application of symbolic methods to frequency arrays.*

S. S. Wilks, Iowa, June, minor in education, *On the distribution of statistics in samples from a normal population of two variables with matched sampling of one variable.*

R. P. Winch, Wisconsin, June, major in physics, minor in mathematics, *Photoelectric properties of silver.*

Yue-Kei Wong, Chicago, December, *Spaces associated with non-modular matrices with application to reciprocals.*

Professor Vannevar Bush has been appointed dean of engineering and vice-president at the Massachusetts Institute of Technology.

Dr. James W. Glover, who has been on leave of absence from the University of Michigan, and who has been president of the Teachers Insurance and Annuity Association of America, has resigned from his position in the Association and will resume his professorship of mathematics at the University of Michigan beginning with the coming academic year.

Assistant Professor Tobias Dantzig, of the University of Maryland, has been promoted to an associate professorship of mathematics.

Guillaume Bigourdan, honorary chief astronomer of the Paris Observatory, and member of the Paris Academy of Sciences in the section of astronomy, died February 28, 1932, at the age of eighty.

Dr. J. C. Glashan, of Ottawa, Canada, died March 14, 1932, at the age of eighty-eight. Dr. Glashan had been a member of the American Mathematical Society since 1891.

D. H. Marshall, professor emeritus of mathematical physics at Queen's University, Kingston, Ontario, died March 14, 1932, at the age of eighty-four.

Dr. James Mercer, F. R. S., fellow and lecturer in mathematics at Christ's College, Cambridge, died February 21, 1932, at the age of forty-nine.

Professor Wilhelm Ostwald, of the University of Leipzig, known for his work in physical chemistry, died April 4, 1932, at the age of seventy-eight.

Professor H. J. Priestley, of the University of Queensland, Brisbane, died February 26, 1932, at the age of forty-eight.

Dr. T. H. Gronwall, associate in physics at Columbia University, died May 9, 1932, at the age of fifty-five. He had been a member of the American Mathematical Society since 1912.

Dr. F. M. Hartmann, dean of the School of Engineering, Cooper Union, died March 28, 1932, at the age of sixty-one. Dean Hartmann was a member of the American Mathematical Society.

Miss Lylah Kryder, instructor in mathematics at Brooklyn College, died March 29, 1932.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

119. Dr. Leo Zippin (National Research Fellow): *On the Rutt-Nöbeling theorem.*

A new and thoroughly independent proof is given that a locally compact continuous curve C, containing two points x and y such that no N points of C separate x and y, contains at least $N+1$ independent arcs xy. The proof is preferred to that recently published by Nöbeling (in vol. 18 of the Fundamenta Mathematicae, of which advance reprints have just been received) because (1) it is thoroughly inductive, although it has been commonly held that no inductive proof is here possible, (2) it is independent of the Menger "n-Bein Satz" to which the Nöbeling argument is reduced, and therefore gives a new proof of this theorem also, (3) it extends the field of the theorem to locally compact spaces, (4) the proof is much simpler certainly than the combined Menger-Nöbeling papers. The extension to locally compact spaces has this peculiar interest that it strikingly facilitates an induction, which in compact spaces might not be suspected. (Received March 17, 1932.)

120. Dr. Oscar Zariski: *A topological theorem on algebroid singularities.*

The intersection of an algebroid singularity, given by its Puiseux expansion $y=y(x)$, with the boundary of the 4-cell $|x|<\text{const.}$, $|y|<\text{const.}$, is stereographically projected into a knot of ordinary space. These knots have been described by K. Brauner, who also gave the generating relations of their fundamental group. That two distinct singularities give rise to distinct knots has been proved by O. Schreier for the particular case in which both knots lie on a torus (singularities of genus 1). The purpose of the present paper is to prove the following general theorem: *If two algebroid singularities are distinct from the algebro-geometric point of view, i.e., if the characteristic numbers which occur in their respective Puiseux expansions are distinct, then they are also topologically distinct, and their fundamental groups are not isomorphic.* (Received March 7, 1932.)

121. Mr. F. G. Dressel: *A generalized boundary value problem for the heat equation.*

Taking the fundamental region as a rectangle, let G_1, G_2 be functions of limited variation along the respective vertical sides of the rectangle, vanishing

at the lower vertices and having the property that $G_i(t)=G_i(t-0)$. We consider the function $\phi_1(x, t)+\phi_2(x, t)$ of the type mentioned in the paper by Dressel and Miles presented at the same meeting of this Society (see abstract 123, below). The integrals ϕ_i are extended along vertical segments which on being allowed to approach the sides of the rectangle lead to a system of Stieltjes integral equations. The solution of this system is a unique function of the class considered, having the desired properties of vanishing along the base of the rectangle and taking on the respective values G_1, G_2 along the vertical sides. The usual treatment is the special case in which the boundary values are the integrals of continuous functions. The problem of the more general plane region is being studied. (Received March 7, 1932.)

122. Professor Antoni Zygmund: *On lacunary trigonometric series.*

A lacunary trigonometric series is defined by (1) $\Sigma(a_k\cos n_kx+b_k\sin n_kx)$, $n_{k+1}/n_k>q>1$. It is proved that if the partial sums of (1) are uniformly bounded below or above on a set E of positive measure, then $\Sigma(a_k^2+b_k^2)$ converges. The infinite products $\Pi(1+a_k\cos n_kx)$ are used systematically in this paper for constructing Fourier-Stieltjes series which converge to zero almost everywhere. (Received March 7, 1932.)

123. Mr. F. G. Dressel and Professor E. R. C. Miles: *A class of solutions of the heat equation.*

Consider the Poisson-Stieltjes integral for the heat equation, $u(x, y)=(1/(2\cdot\pi^{1/2}))\int_a^b(1/(y-\eta)^{1/2})e^{-\mu(\xi)}dF(\xi)$, where $\mu(\xi)=(x-\xi)^2/(4(y-\eta))$ and $F(\xi)$ is a function of limited variation with regular discontinuities. Under slight restrictions as to the manner in which the point (x, y) approaches an interior point x_0 of the interval (a, b) on the characteristic of $y=\eta$, $u(x, y)$ is shown to take on the value $F'(x_0)$ whenever this exists. If $F(t, y)$ denotes the integral from a to t of $u(x, y)$ with respect to x, then $\lim_{y\to\eta}F(t, y)=F(t)-F(a)$. The function $v(x, y)=(1/(2\cdot\pi^{1/2}))\int_a^y(x-x_0)(y-\eta)^{-3/2}\cdot e^{-\mu(x_0)}dF(\eta)$ is analogous to the potential of a double layer, and here it is shown that, according as x approaches x_0 from the right or from the left, $v(x, y)$ will take on the value plus or minus $F'(y-0)$, when this one-sided derivative exists. The function $\phi(x, t)$ defined by integrating $v(x, y)$ from a to t with respect to y has the property that $\lim_{x\to x_0}\phi(x, t)$ is plus $[F(t-0)-F(a)]$ for $x>x_0$ and minus this same value if $x<x_0$. (Received March 7, 1932.)

124. Dr. R. P. Agnew (National Research Fellow): *On summability of double sequences.*

Let $\|a_{mi}\|$ and $\|b_{nj}\|$ be triangular matrices satisfying the conditions (1) for each i, $\lim_{m\to\infty}a_{mi}=0$; for each j, $\lim_{n\to\infty}b_{nj}=0$; (2) for each m, $\sum_{i=0}^{m}|a_{mi}|<K$; for each n, $\sum_{j=0}^{n}|b_{nj}|<K$, where K is a constant; and (3) $\lim_{m,n\to\infty}\sum_{i=0}^{m}\sum_{j=0}^{n}a_{mi}b_{nj}=1$. Corresponding to each double sequence s_{ij}, let $S_{mn}=\sum_{i=0}^{m}\sum_{j=0}^{n}a_{mi}b_{nj}s_{ij}$. We show that if s_{ij} converges to s and if each sufficiently advanced row and column of its transform S_{mn} is bounded, then S_{mn} converges to

s. We show also that if the matrices $\|a_{ms}\|$, $\|b_{nj}\|$ satisfy a certain condition in addition to (1), (2), and (3), and if s_{ij} converges and each sufficiently advanced row and column of S_{mn} is bounded, then each row and column of S_{mn} is bounded. Applications are given. (Received March 25, 1932.)

125. Professor W. A. Hurwitz: *Definition of a field by four postulates.*

The small number of postulates used for the definition of a group in this paper is obtained by means similar to those previously employed by the author. The associative and commutative laws of addition are expressed by one formula; the associative and commutative laws of multiplication and the distributive law, by another formula. Complete independence is verified, even when the set is amplified by postulation of number of elements. (Received March 25, 1932.)

126. Mr. Walter Prenowitz: *Characterization of collineations on the real projective plane.* Preliminary report.

In this paper the following theorem is proved: Any one-to-one point transformation on the real projective plane, which carries the lines of four pencils with non-collinear vertices into lines, is a collineation. (Received March 23, 1932.)

127. Professor I. A. Barnett: *Functional invariants of integro-differential equations.*

The object of this investigation is the study of functional invariants of systems of linear homogeneous integro-differential equations under the Fredholm group. In this paper the single equation

$$y_{tt}(x,t) + \int_0^1 L(x,\eta,t)\, y_t(\eta,t)d\eta + \int_0 M(x,\eta,t)y(\eta,t)d\eta = 0$$

is considered and is subjected to the transformation

$$y(x) = z(x) + \int_0 K(x,\eta)\, z(\eta)\, d\eta.$$

The induced functional group on the coefficient kernels and their derivatives is then found both in the finite and infinitesimal forms. From this group are deduced the functional equations which the invariants satisfy. A number of functional invariants may be derived by writing the integro-differential equation in a canonical form in which the term involving the first derivative is missing. (Received March 17, 1932.)

128. Mr. Gordon Fuller: *On the invariant character of a set of differential equations.*

This paper deals with a set of equations (the subscripts denote differentiation): $\delta_x+\gamma_y+\alpha_z-\alpha/2=0$, $\gamma_z-\delta_y+\beta_z-\beta/2=0$, $\beta_z+\alpha_y-\gamma_x-\gamma/2=0$, $\alpha_z-\beta_y-\delta_z-\delta/2=0$, which belong to a type that has recently gained prominence in physics. Equations of this type can be proved to be invariant under transformations of coordinates, although they lack symmetry with respect to x, y, and z. The system is treated by introducing a set of four new dependent vari-

ables, three of which may be considered as components of a vector. The fourth, which does not transform in any orthodox way, is eliminated by differentiation. The non-linear set of second order equations thus obtained is then written in a form in which the invariance is easily recognizable. A transformation on a vector with components u, v, w involving their partials with respect to x, y, z is used. (Received March 17, 1932.)

129. Professor Tibor Radó: *On approximately conformal maps of surfaces.*

Given a surface $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, such that $EG-F^2>0$, it is possible to find a conformal map of the surface, that is to say a map such that $E=G$, $F=0$. If $EG-F^2$ has zeros, then a conformal map does not generally exist. In his work on the problem of Plateau, the author used *approximately conformal maps*, that is to say maps for which $\iint(E^{1/2}-G^{1/2})^2$ and $\iint|F|$ are arbitrarily small. The object of this paper is to develop an existence theorem for such maps and applications of this theorem to the problem of Plateau. (Received March 17, 1932.)

130. Mr. D. S. Nathan: *Sphere geometry and the conformal group in function space.*

This paper develops the sphere geometry of euclidean function space R_x as the analogue of the elementary sphere geometry of euclidean n-space. The element is the sphere in R_x, namely,

$$\sigma \int_0^1 f^2(y)\,dy - 2\int_0^1 \phi(y) f(y)\,dy + \omega = 0,$$

where $f(x)$ is a continuous function on the interval $0 \leqq x \leqq 1$ representing the variable point, $\phi(x)$ is a given continuous function on this interval, and σ, ω are given real numbers on the interval $0 \leqq \sigma, \omega \leqq 1$. We call $\phi(x)$, σ, ω the homogeneous sphere coordinates. Conformal transformations are defined as those linear functional transformations in $\phi(x)$, σ, ω which take spheres of zero radius into spheres of zero radius. They also preserve orthogonality of spheres. A conformal transformation whose Fredholm determinant is not zero is called non-singular. The totality of non-singular conformal transformations forms a group in the sense that the succession of two transformations gives rise to another of the same kind. The infinitesimal conformal transformations are found to be identical with the "regular infinitesimal conformal transformations in homogeneous coordinates" which Kowalewski obtained as the angle-preserving transformations in R_x. The one-parameter group of non-singular conformal transformations generated by an infinitesimal conformal transformation is determined. (Received March 17, 1932.)

131. Dr. A. T. Craig: *Some properties of normally correlated variables.*

Let $t_1, t_2, \cdots, t_n$ be n variables each subject to a Gaussian law of probability. Let t_i, $(i=1, 2, \cdots, n-1)$, be normally correlated with t_{i+1} with the coefficient of correlation $r=r_i$. No assumptions are made regarding the correlation between the variables other than that of the correlation between them in

adjacent pairs. In the present paper, a set of functions $\psi_{ij}(t_i, t_j)$, $i+1<j\leqq n$, is determined such that $\psi_{ij}(t_i, t_j)$ is the correlation function of the characters t_i and t_j wherein t_i is possessed by one individual and t_j is possessed by a second individual, the two individuals having been paired identically as to the character t_{j-1}. A correlation coefficient $r=r_i\cdots r_{j-1}$ measures the correlation between the two variables when paired in this manner. (Received March 18, 1932.)

132. Dr. Selby Robinson: *Certain cardinal numbers associated with a point in a general topological space.*

This is a study of equalities and inequalities involving certain numbers associated with a point p. One of these numbers is the power of the least complete family of neighborhoods of p, and another is the power of the least family of neighborhoods whose product is not a neighborhood of p. The equality of these two cardinals implies the equality of all the cardinals considered, and is a necessary and sufficient condition that p have a monotone complete family of neighborhoods. An equivalence which was previously proved by E. W. Chittenden and the author (this Bulletin, vol. 37, p. 628) for spaces V_ω, is extended in a modified form to spaces in which every point has a monotone complete family of neighborhoods. (Received March 18, 1932.)

133. Miss I. M. Schottenfels: *On the interchange of limit and integral for classes of functions in real, complex, and quaternionic numbers.*

This paper develops theorems for the general case of the interchange of limit and integral for classes of functions of real, complex and quaternionic numbers, and incidentally shows that the theorems in the papers of R. L. Jeffery and T. H. Hildebrandt on this subject are special instances of the theorems in this paper. In conclusion it is shown that limits and the Lebesgue, Riemann, and Riesz integrals are interchangeable, and the identity of these integrals is established, as was recently mentioned in a paper by W. M. Whyburn. (Received March 9, 1932.)

134. Professor G. A. Bliss and Dr. M. R. Hestenes: *Sufficient conditions for a problem of Mayer in the calculus of variations.*

In this paper sufficient conditions for a minimum are deduced for a modification of the classical problem of Mayer in the calculus of variations. The problem is that of finding in the class of arcs $y_i=y_i(x)$ $(i=1, \cdots, n; x_1\leqq x\leqq x_2)$ satisfying the differential equations and end conditions $\phi_\alpha(x, y, y')=0$, $(\alpha=1, \cdots, m)$, $\psi_\rho[x_1, y(x_1), x_2, y(x_2)]=0$ $(\rho=1, \cdots, 2n+1)$ one which minimizes a function $g[x_1, y(x_1), x_2, y(x_2)]$. The classical problem of Mayer is the special case for which the function g to be minimized is the end value $y_1(x_2)$ and the conditions $\psi_\rho=0$ specify the other end values of the variables $x, y_1, \cdots, y_n$. This problem is equivalent to one formulated by Bolza for which sufficiency theorems have been deduced by Morse and Bliss. In both cases, however, the proofs depend upon assumptions which prevent the problem of Bolza from being equivalent

to the most general problem in the Mayer form. One of the most important features of this paper is the use of an $(n+1)$-dimensional field in $(x, y_1, \cdots, y_n)$-space rather than the n-dimensional fields which have been used hitherto by Kneser, Egorov, and Larew for the classical problem of Mayer and which do not seem to be applicable here. It is interesting to note that the auxiliary minimum problem associated with the second variation is abnormal. (Received April 8, 1932.)

135. Dr. M. R. Hestenes: *Sufficient conditions for a problem of Mayer with variable end-points.*

The problem considered in this paper is that of finding in the class of arcs $y_i=y_i(x)$, $(i=1, \cdots, n;\ x_1 \leqq x \leqq x_2)$, satisfying the differential equations and end-conditions $\phi_\alpha(x, y, y')=0$ $(\alpha=1, \cdots, m<n)$, $\psi_\rho[x_1, y(x_1), x_2, y(x_2)]=0$ $(\rho=1, \cdots, p<2n+1)$ one which minimizes a function $g[x_1, y(x_1), x_2, y(x_2)]$. This is the problem of Mayer with variable end-points as formulated by Bliss (Transactions of this Society, vol. 19 (1918), p. 305). Sufficient conditions by Bliss and Hestenes for the case when $p=2n+1$ are applied to establish similar conditions in the general case. The proof for the case when $p<2n+1$ is different from that for the case $p=2n+1$, but makes use of the sufficiency theorem for the latter case. The method used is related to that used by Bliss for the problem of Bolza (Annals of Mathematics, (2), vol. 33 (1932)). Theorems on normality are given, and the necessary condition of Mayer for variable end-point problems is given a new form. The results obtained are equally applicable to the problem of Bolza considered as a problem of Mayer with variable end-points but the proofs hitherto given by Morse and Bliss for the problem of Bolza do not apply to the general case of the problem of Mayer. (Received April 8, 1932.)

136. Professor W. H. Durfee: *Summation factors for double series.*

The double series considered in this paper are of the type $J(z, w)=\sum_{m,n=1}^{\infty} a_{m,n}z^{f(m)}w^{g(n)}$, where $\sum_{m,n=1}^{\infty}a_{m,n}$ is summable by some Cesàro mean with the value s and has bounded partial sums, z and w are complex variables, and $f(m)$ and $g(n)$ are logarithmico-exponential functions. Sufficient conditions on $f(m)$, $g(n)$ are given so that the series shall converge for all values of z and w within the unit circle, and so that $\lim J(z, w)=s$ as z and w independently approach unity from within the unit circle. (Received March 15, 1932.)

137. Mr. Harry Matison: *On the nature of the solutions of certain types of integro-differential equations.*

Equations of the type $\partial_z(x, t)/\partial t=F[z(r, s);\ x, t]$ are considered. Under suitable hypotheses involving the existence and continuity of the Fréchet differential of the right hand side, the existence of the derivative of the solution with respect to x is established. (Received March 16, 1932.)

138. Mr. R. S. Martin: *Contribution to the theory of ana'ytic functions in abstract spaces.* Paper II.

In a preliminary report under the same title (this Bulletin, Nov., 1931, p. 824, abstract 371) the author announced results on the properties of analytic correspondences in Banach spaces with closure under multiplication by numbers of a ring. The special cases of closure under the fields of real and complex numbers are examined in more detail. The equivalence of various possible definitions of abstract polynomials is shown to depend on whether or not the field contains a complex quantity. Incidentally an elegant treatment of the Fréchet theory of abstract polynomials is included. These results are employed to obtain further development of analytic function theory in abstract spaces. (Received March 17, 1932.)

139. Professor M. A. Basoco: *On the trigonometric developments of certain doubly periodic functions of the second kind.*

In this paper we are primarily concerned with deriving the explicit arithmetized Fourier series expansions for the functions

$$\phi_{\alpha\beta\gamma}(x, y) \equiv \theta_1'^2 [\theta_\alpha^2 (x+y)/\theta_\beta^2 (x)\theta_\gamma^2 (y)]$$

where $\theta_\mu(z)$ are the elliptic theta functions of Jacobi, (x, y) are independent complex variables and (α, β, γ) are certain sixteen triads out of the possible sixty-four which can be selected from the numbers 0, 1, 2, 3. These functions belong to the class of doubly periodic functions of the second kind (in the sense of Hermite), and hence can be treated by the methods of Teixeira given in the Journal für Mathematik, vol. 125. A generalization of these results lead to certain formulas of decomposition which when applied to the functions $\phi_{\alpha\beta\gamma}(x, y)$ yield the desired expansions. These developments may be used to obtain formulas of the Liouville type involving arbitrary numerical functions (e.g. see E. T. Bell, Transactions of this Society, vol. 22 (1921); Colloquium series, VII, p. 88, 1927). Finally, the developments obtained can be made to yield the expansions of certain functions of the third kind. (Received March 15, 1932.)

140. Dr. I. J. Schoenberg: *Concerning the theory of systems of linear inequalities.*

This paper contains a geometric exposition of the fundamental results of Minkowski and Farkas with the additions given by Haar, who presented an elegant geometric treatment of the subject (Szeged Acta, vol. 2 (1924), fasc. 1, pp. 1–14) based, however, on a rather restrictive assumption which asks that the set of points in n-space which are solutions of the given system of linear inequalities in n variables has interior points. In the present paper this assumption is not made. Moreover some of the results have been extended to cases which have not been treated before. This paper, together with the paper presented by the author at the New Orleans annual meeting, will appear in the Transactions of this Society. (Received March 15, 1932.)

141. Professor W. A. Manning: *On primitive groups containing circular permutations of degree p^a or $2p^a$.*

If a non-alternating primitive group of degree n contains a circular permutation of prime degree m, then $n \leqq m+2$. This was proved by C. Jordan. In the present paper it is shown that the theorem remains true if $m=p^a$ or $2p^a$ (p prime). (Received March 18, 1932.)

142. Professor J. S. Turner: *Quadratic cycles.*

The quadratic cycles are of the type $d-x_1^2=r_0r_1$, $d-x_2^2=r_1r_2, \cdots$, $d-x_n^2=r_nr_0$, where x, r, and d are positive integers, d not a perfect square, obtained when $(\sqrt{d}-x)/r$ is converted into a continued fraction. The results obtained in this paper include (a) necessary and sufficient conditions that the cycle may be symmetric, (b) the number of symmetric cycles for a given d, (c) unless $r_0=r_1$ and $d=2x_1^2$ or $5x_1^2$, the cycle contains an $r<\sqrt{d}/2$. (Received March 7, 1932.)

143. Mr. C. R. Worth: *Subvarieties of a field.*

The postulates for a field given by Dickson (Transactions of this Society, 1905, p. 198) are shown to be completely independent in the sense of E. H. Moore for both finite and infinite classes when closure is assumed for both addition and multiplication. The set obtained by adding the postulate of commutativity of addition is shown to give consistent number systems in almost all cases. Of the 2^{13} number systems arising from a modification of a postulate system due to Huntington (ibid., p. 181) the consistency of all except approximately 190 has been settled. The purpose of this paper is not to show the complete independence of the postulate systems under consideration, but to determine the consistent subvarieties of a field. (Received March 15, 1932.)

144. Professor G. D. Birkhoff and Dr. W. J. Trjitzinsky: *Analytic theory of singular difference equations.*

In this paper the authors develop the analytic theory of linear difference equations when no restrictions whatsoever are placed on the formal series solutions. The fundamental result is as follows. Depending on the exponential factors in the formal series the complete neighborhood of infinity is divided into a finite number of regions, separated by curves with a limiting direction at infinity; if R is such a region, there exists a fundamental set of solutions with elements analytic and asymptotic to the formal series in R. The converse and the related Riemann problems are also solved. The paper is based, on one hand, on the earlier papers of Birkhoff on difference equations; on the other hand, essentially new methods are used involving factorization and "group summation." This work will appear in the Acta Mathematica. (Received April 9, 1932.)

145. Dr. Walter Bartky and Professor R. W. Barnard: *Note on the solution of normal systems of linear differential equations with constant coefficients.*

A fundamental set of solutions of the differential equations $dx_i/dt=\Sigma_j\kappa_{ij}x_j$, $(i, j=1, \cdots, n)$, where $\kappa=\|\kappa_{ij}\|$ is a matrix of constants, is $\phi(\kappa)=\Sigma_i e^{\alpha_i t}\cdot[\delta+(\kappa-\alpha_i\delta)t/1!+\cdots+(\kappa-\alpha_i\delta)^{m_i-1}t^{m_i-1}/(m_i-1)!]\Pi_{j\neq i}(\kappa-\alpha_j\delta)^{m_j}$. The α_i are characteristic values for κ of multiplicity m_i and δ is the identity matrix. That $\phi(\kappa)$ is a solution is readily verified by substitution and noting that κ satisfies its characteristic equation. Since the matrix $\phi(\kappa)$ is a polynomial in κ, its characteristic values are $\phi(\alpha_i)=e^{\alpha_i t}\Pi_{j\neq i}(\alpha_i-\alpha_j)^{m_j}\neq 0$ and, therefore, $\phi(\kappa)$ is a fundamental set. (Received April 9, 1932.)

146. Professor Tibor Radó: *On the second variation of the area-integral.*

Schwarz has already constructed examples of minimal surfaces which do not have a smallest area. The object of this paper is to call attention to examples which can be discussed in a perfectly elementary way, and also to point out that certain uniqueness theorems, proved by the author in previous papers, lead to sufficient conditions for an absolute minimum. (Received March 16, 1932.)

147. Professor O. E. Glenn: *Sketch of an important solvable problem in formal modular covariants.* Preliminary report.

The author proved in 1915 that the binary form with arbitrary coefficients,

$$f_m=a_0x^m+a_1x^{m-1}y+\cdots+a_my^m,$$

is reducible, modulo 2, in terms of covariants of f_m of orders <4, together with the known universal covariants of the total group, (mod 2). It follows that the system of covariants, modulo 2, of any f_m, $m<8$, will be known when the joint system of (f_1, f_2, f_3) has been determined. The object of this investigation is to determine the latter simultaneous system. (Received March 16, 1932.)

148. Professor A. D. Michal and Mr. J. L. Botsford: *The new Einstein geometry and some of its extensions.*

We consider an n-dimensional Riemannian space R_n, to each point of which is associated an m-dimensional linear vector space $V_m(m\geqq n)$. Correspondences between vectors in V_m and vectors in the associated tangent space are defined. Geometries involving a König linear connection and a set of normal orthogonal exceptional vectors are developed. Several theorems on the differential invariants of such geometries are proved. The results, for $m=5$, $n=4$, include those considered recently by Einstein and Mayer (Sitzungsberichte der Preussischen Akademie der Wissenschaften, December, 1931, pp. 541–557). (Received March 17, 1932.)

149. Professor H. A. Simmons: *Further generalizations of certain classes of extreme numbers relative to certain symmetric equations in n reciprocals.*

In two papers which were presented respectively at the Chicago and New Orleans meetings of the Society in 1931, the author identified certain rather

general classes of maximum numbers and minimum numbers relative to certain elementary symmetric equations in n reciprocals. In this paper considerable extensions of the previous results are made by dealing with certain symmetric equations that are not elementary symmetric. (Received March 17, 1932.)

150. Professor R. L. Jeffery: *Non-absolutely convergent integrals with respect to a function of bounded variation.*

Let $\alpha(x)$ be a function of bounded variation on (a, b), and $F(x)$ a function with discontinuities of the first kind only. The derivative of F with respect to α, and the determination of F when the derivative is given and is finite at each point is studied. The latter problem leads to a theory of non-absolutely convergent integrals with respect to a function of bounded variation analogous to the Denjoy-Khintchine integral with respect to the variable x. Some of the results are: (1) If the derivative of F with respect to α, $D_\alpha F$, exists at each point it is measurable relative to α. (2) If $D_\alpha F$ is finite at each point, then it is Denjoy-Khintchine integrable with respect to α and its integral is equal to $F(b)-F(a)$. (3) If the function $F(x)$ is the Denjoy-Khintchine integral with respect to α of the function $f(x)$, then the approximate derivative of $F(x)$ with respect to α is equal to $f(x)$ except for at most a set of α-measure zero. (4) A discussion of the properties of Denjoy-Khintchine integrals with respect to α, including a formula for integration by parts, and the second mean value theorem for integrals. (Received April 1, 1932.)

151. Dr. A. E. Currier: *The variable end point problem of the calculus of variations including a generalization of the classical Jacobi conditions.*

Let H and H' be two transverse families of extremals in m-space $(z) = (z_1, \cdots, z_m)$. Let g be an extremal belonging to both H and H'. Let M and M' be transverse manifolds of H and H' respectively. Let V and V' be the Hilbert integrals associated with H and H' respectively. We are concerned with a variable end point problem in which the end points vary on M and M' respectively. We define a new invariant function $I(z, w) = \Sigma_{i,j=1}^{m} [V_{z_i z_j} - V'_{z_i z_j}] w_i w_j$, where the partial derivatives of the Hilbert integrals are evaluated at the point (z) on g, and (w) is arbitrary. Associated with each point (z) of g we obtain a set of necessary conditions for a relative minimum along g and also a set of sufficient conditions. These conditions involve the number of focal points of H on the segment of g between M and (z), the number of focal points of H' on the segment of g between (z) and M', and the type numbers of the quadratic form $I(z, w)$. In case (z) lies outside the segment of g between M and M' our conditions give the generalization for m-dimensional space of the classical Jacobi conditions of the problem in the plane. (Received April 6, 1932.)

152. Professor Clifford Bell: *Alternant surfaces.*

The surfaces obtained from the third order alternant, $A = |\alpha^m \beta^n \gamma^p|$, $m < n < p$, by setting $\beta/\alpha = x$, $\gamma/\alpha = y$ and $A/[(\beta\gamma)^m \alpha^{m+n+p}] = z$, are shown to

fall into four types. Three concurrent lines are common to all the surfaces along which z vanishes, and their intersection is a common minimax point. By means of these surfaces it is possible to determine the existence, or non-existence, of values of α, β, γ, no two of which are equal, such that A vanishes. (Received April 18, 1932.)

153. Professor Clifford Bell: *On some properties of polygons related to the cuspidal cubic and a line.*

The n-sided polygons studied are those having $n-1$ vertices on, and n sides tangent to, the cuspidal cubic, the remaining vertex being on a given line. The polygons, so related to the cubic and a given line, are shown to be perspectively related for certain positions of the line. Other projective properties of the polygons are developed, together with theorems concerned with the lines through their corresponding vertices. (Received April 18, 1932.)

154. Dr. H. B. Curry: *Some properties of equality in combinatory logic.*

This paper deals with properties which are of the same general type as those considered in a previous paper, *Apparent variables from the standpoint of combinatory logic* (see abstract 37–1–60), and which result from the introduction of certain new axioms for equality and implication. The results include certain formulas expressing formally the properties of equality previously established informally. The principal theorem, however, amounts to a justification of what may be called the postulational method of proof, i.e., the method whereby, wishing to establish that a certain conclusion is a formal consequence of certain premises (both premises and conclusion being propositional functions of n variables), one argues as follows: Let $x_1, x_2, \cdots, x_n$ be arbitrary entities such that the premises are satisfied, then for these x's the conclusion is also satisfied. The justification consists in the exhibition of a general method whereby, given any valid argument of the above nature, the same may be transformed into a demonstration, within the formalism of combinatory logic, of a formula whose interpretation is the fact required to be proved. (Received April 21, 1932.)

155. Mr. B. F. Groat: *Misuse of postulations in theory of viscosity.*

Studies in Newtonian similitude (this Bulletin, vol. 25, p. 52; vol. 36, pp. 151–194; Proceedings of the American Society of Civil Engineers, Aug. 1931, pp. 938–989; The Journal of Engineering Education, Jan. 1932, pp. 379–383) disclose misuse of postulation in the beginnings of the kinetic theory of viscosity. Error is first indicated by the axiom: no solution in mathematical physics is valid if an assumption is necessary to it (discloses similar error in hydrodynamics, Proceedings of the American Society of Civil Engineers, p. 959). Solutions result only from existing theory. Assumptions are precluded, or, if of undeniable force, they postulate new theories, not the old. Empiricism may serve in some engineering problems, but not in a formal science based upon

"a priori" postulates which agree with experience throughout the region of practical observation. This mathematical philosophy is confirmed by mathematical analysis showing that Maxwell's assumed free-path distribution, creates fallacies appearing in all books treating the subject. When expunged viscosity theory will become valuable. Resulting errors in physical constants also need correction. It is remarkable that Newton and Maxwell each made an error in viscosity,—Newton by misusing his correct postulation; Maxwell by making an empirical postulation. (Received May 4, 1932.)

156. Professor R. D. Carmichael: *Summation of Functions of a Complex Variable.*

The main object of this paper is to develop a theory of the principal solutions of the equations $F(x+\omega)-F(x)=\omega\phi(x)$, $G(x+\omega)+G(x)=2\phi(x)$. In part I a definition of principal sums is employed which is closely related to that of Nörlund (Acta Mathematica, vol. 44 (1922), pp. 71–211) and the theory is developed for the case of functions $\phi(x)$ having a certain defined asymptotic character at infinity. In part II generalized sum formulas of the Euler-Maclaurin type are introduced and are employed in constructing principal sums of functions of exponential type; and these sums are developed in Fourier series by a direct method. In part III necessary and sufficient conditions are given for the expansion of functions in series of Bernoulli-Hurwitz functions, including the special case of Bernoulli polynomials. (Received May 6, 1932.)

157. Dr. Hassler Whitney: *On 2-congruent graphs.*

It is shown that if there is a 1-1 correspondence between the arcs of the graphs G and G' such that if a set of arcs of one graph form a circuit (or a subgraph of nullity 0) then the corresponding arcs of the other do also, then G can be carried into G' by operations of the two following types. (1) The graph is broken at a single vertex into two pieces, or two pieces are joined at a vertex. (2) If the graph is composed of two connected graphs H_1 and H_2 which have just the two vertices a and b in common, then H_1 is turned around so that its arcs which were formerly on a (b) are now on b (a). By definition, G and G' are 2-congruent. (Received May 7, 1932.)

158. Dr. H. L. Garabedian: *Note on a theorem due to Bromwich.*

The theorem of Bromwich under consideration gives sufficient conditions in order that a definition of summability with infinite matrix of reference shall include Cesàro summability of order k. The present paper reestablishes this theorem by a more direct and shorter method than that used by Bromwich. Moreover, this proof affords a method of exhibiting a k-fold summability with infinite matrix of reference, analogous to well known definitions of summability with finite matrices of reference which make use of repeated means, for any convergence factor which satisfies the conditions of the theorem under discussion. (Received May 10, 1932.)

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

On account of the fact that the Summer Meeting of the Society in 1932 will be held on the Pacific Coast, the meetings that were to have been held at Seattle (June, 1932) and at Los Angeles (November, 1932) will be omitted.

LOS ANGELES, CALIFORNIA, SUMMER MEETING AND COLLOQUIUM, August 30-September 2, 1932.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than July 26. Abstracts received by July 7 will appear in the July issue, or an earlier issue, of this BULLETIN. Professor J. F. Ritt will deliver a series of colloquium lectures on *Differential equations from the algebraic standpoint*. By invitation of the program committee, Professor D. N. Lehmer will speak on *The continued fraction representing cubic and higher irrationalities*, and Professor Tibor Radó will speak on *Recent work in the problem of Plateau*.

NEW YORK CITY, October 29, 1932.

Abstracts must be in the hands of Associate Secretary Tomlinson Fort, 501 West 116th St., New York City, not later than October 8. In order to be printed in the BULLETIN in advance of the meeting, abstracts must be in the hands of the Associate Secretary not later than September 7. By invitation of the program committee, Professor Jesse Douglas will speak on *The problem of Plateau*.

AMES, IOWA, November 25-26, 1932.

Abstracts are due by November 3, 1932.

ATLANTIC CITY, NEW JERSEY, ANNUAL MEETING, December 27-30, 1932.

Abstracts are due by November 30, 1932.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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